FUZZY OPTIMIZATION OF FOOT-TRAJECTORY PROFILES IN WALKING MACHINES

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Abstract

Leg dynamics obstructs the trajectory following at high speed, thus being responsible of tracking errors and system instability. The election of trapezoidal or parabolic foot velocity profiles can deal with leg dynamic effects. However, the effects of leg dynamics are different for each foot trajectory. Therefore, to optimize foot speed, the foot velocity profile should adjust to leg dynamics for each trajectory in the whole leg workspace. The herein proposed method for trajectory generation is suitable for achieving accurate, smooth and fast foot movements, increasing leg speed as much as leg dynamics allows for each trajectory.

Keywords: Walking Machines, Legged Locomotion, Fuzzy-Logic Based Systems, Trajectory Generation.

1. INTRODUCTION

Detractors of legged robots usually point out the machine speed as one of the major shortcomings of these vehicles. Optimization of speed in robot manipulators has been extensively studied in the last two decades [1,9,10]. Similar methods have been developed to find the minimum-time control of serial manipulators along specified paths with actuator torque limitations. The minimum-time control theory assumes that a perfectly accurate manipulator model is available and that there are no external disturbances. In practice, however, it is not possible to obtain such an ideal model.

In this article the authors propose a method to find optimal speed of trajectory tracking for the legs of a walking machine. The proposed approach is an improvement of previous work on this subject [5], where trapezoidal foot-velocity profiles were adjusted to leg dynamics using a fuzzy acceleration tuning approach. However, trapezoidal velocity profiles are not smooth enough, due to the discontinuity in acceleration. Therefore, in this work trapezoidal foot profiles with parabolic blends are used instead [2] (see Figure 1), improving the fuzzy acceleration tuning algorithm proposed in [5]. The SILO4 walking robot is used as a testbed for testing the developed algorithms [3]. The method proposed is computationally efficient and its real time implementation for on-line path generation is proved. The proposed algorithm does not include a mathematical model of the robot leg to avoid errors due to mathematical simplification. It includes the experimental observation of real dynamic effects existing during the leg motion instead. The dynamics affecting the leg motion were carefully analyzed in previous work [4] and results are introduced in this algorithm as fuzzy rules reflecting the existing dynamics. Since rules are imposed on the foot motion, the computational complexity of the algorithm is independent on the number of degrees of freedom of the robot leg.

The application of fuzzy set theory is specially recommended when a mathematical model of the system is not available. In the authors’ opinion, it provides an efficient representation of the real dynamic effects over the system motion. Fuzzy rules also provide a good representation of parameter uncertainties existing on real machines.

For a better understanding of this work, Section 2 states the problem, which is solved by fuzzy rules. Experimental results are reported in Section 3, and finally conclusions are reported in section 4.

2. AUTOMATIC FOOT ACCELERATION TUNING

The effect of leg dynamics critically limits foot performance. An earlier work analyzing the SILO4 leg dynamics showed that the relevant dynamics affecting the motion of such a 3-dof articulated leg are inertial effects over the first joint motion and gravitational effects over the second and third joints respectively [4]. This means that leg dynamics could prevent from
following the reference during fast horizontal motions and when raising the foot at high speed. The same effect could affect the downward movement of the leg if it is bearing the robot’s weight. The constant perturbation of gravity added to non-linear frictional and backlash effects could produce oscillations at the beginning of the upward movement if the requested speed is high.

Finding an accurate mathematical model of such dynamic effects over the leg motion is unavoidable. Fuzzy theory is an adequate tool for solving nonlinear system problems where a mathematical model is absent [11]. We employed this soft computing technique to introduce the dynamics affecting the leg motion into a foot-acceleration tuning algorithm, which provided the best acceleration value of the foot for each trajectory. Fuzzy foot-acceleration tuning as a function of the trajectory parameters and leg dynamics could be an adequate technique for improving performance in legged locomotion. Figure 1 shows position, velocity and acceleration profiles used for on-line trajectory generation. Foot acceleration will be tuned considering the following experimental requirements:

1. It is necessary to increase foot acceleration for short trajectories to obtain higher foot speeds.
2. Foot acceleration could be moderate when trajectories are large enough.
3. Foot acceleration should be decreased for vertical movements of the foot to avoid undesired dynamic effects.
4. Foot acceleration should be decreased if the trajectory is close to the drive limits.

The problem of finding the best value of foot acceleration for a given foot trajectory is overcome by using a Mamdani fuzzy inference system [6]. Three input linguistic variables define foot trajectory, which are: desired average foot speed \(v_a\), distance from initial to final position \(s\), and relative z-increment \(\Delta z_{rel}\), which is the ratio between z increment and distance traveled for a given trajectory, that is:

\[
\Delta z_{rel} = \frac{\|z(t_f) - z(t_i)\|}{s} \leq 1
\]  

(1)

The output variable of the fuzzy inference system is the foot acceleration, which is needed for foot velocity profile determination and thus foot trajectory generation.

Input and output fuzzy variables are represented by fuzzy sets (for example, distance is \(BIG\), or foot speed is \(SMALL\)), and the degree of membership of each variable \(\mu(x)\) for the variable \(x\) to the fuzzy sets is given by membership functions.

In this work guidelines on fuzzy controller design have been followed [7]. They state that the inference map shape matches the shape of membership functions of the input variables, provided that membership functions are normal, symmetrical and overlapped by pairs, and the membership functions defined over the output variables have the same area. Taking this into consideration, the following assumptions have been made in order to design the fuzzy system:

1. Let us assume that the relative z increment in a trajectory, \(\Delta z_{rel}\), is represented by two fuzzy sets \{\(SMALL\), \(BIG\)\}. Membership functions of this input variable are trapezoidal and are shown in Figure 2a where the abscissa is the value of the relative z increment and the ordinate \(\mu(\Delta z_{rel})\) is the degree of membership. The relationship between \(\Delta z_{rel}\) and \(a_{foot}\) requires a negative inclination of the resulting inference map along this variable edge, what will be expressed by means of fuzzy rules.

2. Trajectory distance \(s\) is also represented by the same fuzzy sets \{\(SMALL\), \(BIG\)\}. Two trapezoidal distance membership functions are shown in Figure 2b, where the abscissa is the value of the trajectory distance and the ordinate \(\mu(s)\) is the degree of membership. Their limit values were obtained experimentally for the SILO4 leg workspace, where the maximum linear distance for a trajectory is 700 mm.

3. Average foot speed \(v_a\) is represented by the same fuzzy sets as the first two variables. However, membership functions are parabolic rather than trapezoidal (see Figure 2c) just to adjust to the relationship between acceleration and velocity in a trajectory profile. Their limit values were found experimentally for the SILO4 leg example. The average foot speed is limited to 400 mm/s due to limitation of joint drives.

4. The output of this fuzzy inference system is the foot acceleration \(a_{foot}\), which is represented by four fuzzy sets \{\(SMALL\), \(MEDIUM-SMALL\), \(MEDIUM-
BIG, BIG}, and membership functions are shown in Figure 2d. These membership functions are triangular, representing SMALL foot accelerations for long trajectories without any z increment. MEDIUM-SMALL acceleration values for trajectories having a big z increment, MEDIUM-BIG accelerations for long trajectories with high speed, and BIG accelerations for short trajectories with high-speed values. The limit values of membership functions are obtained experimentally for the SILO4 leg example.

Hence, the inference mechanism is based on the following five rules, which represent the fuzzy dependence of foot acceleration on foot speed, trajectory distance and relative z increment:

1. If \( v_m \) is SMALL and \( s \) is SMALL and \( \Delta z_{rel} \) is SMALL then \( a_{foot} \) is MEDIUM-BIG
2. If \( v_m \) is SMALL and \( s \) is BIG and \( \Delta z_{rel} \) is SMALL then \( a_{foot} \) is SMALL
3. If \( v_m \) is BIG and \( s \) is SMALL and \( \Delta z_{rel} \) is SMALL then \( a_{foot} \) is BIG
4. If \( v_m \) is BIG and \( s \) is BIG and \( \Delta z_{rel} \) is SMALL then \( a_{foot} \) is MEDIUM-BIG
5. If \( \Delta z_{rel} \) is BIG then \( a_{foot} \) is MEDIUM-SMALL

MATLAB and its Fuzzy Toolbox were used to solve the fuzzy problem, where \( prod \) represents the and method and implication, \( max \) represents aggregation, and \( centroid \) is used for defuzzification. Once the foot acceleration function has been obtained, optimization methods for real-time implementation of the fuzzy reasoning process can be used [8].

3. EXPERIMENTAL RESULTS

Different experiments using the SILO4 leg have been conducted to show the improvement on straight-line trajectory execution by foot acceleration tuning. One experiment shows the effect of foot acceleration tuning when executing straight-line trajectories of several lengths. Figure 3 illustrates this experiment, depicting the maximum achievable average foot speed for different trajectory distances. Every trajectory was parallel to the leg’s x-axis and for \( y = 215 \text{ mm} \) and \( z = -250 \text{ mm} \). Each thin curve in this graph represents the maximum average foot speed that could be reached with a foot velocity profile without acceleration tuning, provided that no dynamics perturbs the motion. If leg dynamics are considered, some high acceleration trajectories will not be possible to perform because oscillations and non-desired effects will appear as shown in Figure 5. A foot acceleration of 600 mm/s\(^2\) should be used for every trajectory as a conservative value that avoids dynamic effects. The thick curve in Figure 3 represents the maximum achievable foot speed during the same trajectories using foot acceleration tuning, which takes leg dynamics into account. Hence, the acceleration tuning increases foot acceleration if short trajectories are to be performed, until drive limits are found, and it decreases foot acceleration for long ones, where high acceleration is not requested. Thus, foot acceleration tuning finds the acceleration values that provide higher foot speeds, avoiding the use of very high acceleration values that could impose oscillatory behavior. Figure 4 illustrates velocity profiles of two trajectories of different length, \( s(t) \), executed by the leg with and without acceleration tuning at its maximum achievable foot speed (solid and dashed lines respectively). Figure 4a represents a short trajectory, where average speed is clearly improved when using acceleration tuning. Figure 4b shows the same effect for a larger trajectory.
During this experiment, trajectory behavior was the same within different z-component planes, and was only worse for cases where the two last links of the leg are aligned (singularity) due to the excessive increase in internal speed during acceleration time of the second and third joints. However, this situation always appears, whether automatically tuning the acceleration or not, and it implies a reduction of foot acceleration to 600 mm/s², and thus every maximum foot speed value inside this plane of the leg workspace becomes very limited. Average foot speed values as well as distance traveled are listed in Table I for each trajectory of the experiment in Figure 3. The improvement offered by the acceleration tuning algorithm with respect to a conservative acceleration of 600 mm/s² is relevant for short trajectories ($s < 350$ mm) and acceptable for long ones (See Table I). As can be observed from Figure 4, higher average foot speeds can be achieved using the acceleration tuning approach. Table I reveals that the acceleration tuning method increases the average foot speed by 10 to 100 percent over the maximum achievable speed when acceleration tuning is not used.

### 4. CONCLUSIONS

The work presented in this article focuses on optimizing the average leg speed of a walking robot. Acceleration of the velocity profile has been targeted as the proper magnitude to be optimized.

To avoid problems stemming from the robot’s parameter uncertainties, fuzzy techniques have been used. For this purpose, fuzzy rules are defined based on experiments, and the optimal acceleration for every given trajectory is found. A simple Mamdani fuzzy inference system is used to compute the required acceleration. It is based on five rules using three linguistic variables.

Some experiments have been carried out to validate the algorithm. These experiments concluded that the foot acceleration tuning method finds the acceleration values that provide fast and smooth foot trajectories, avoiding perturbing effects due to leg dynamics. This study reveals that, depending on the distance traveled, the acceleration tuning method increases average foot speed by 10 to 100 percent over the maximum achievable speed when acceleration tuning is not used.

### Table I. Comparison of maximum achievable average foot speeds. (*) means actuator limit found.

<table>
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<tr>
<th>$s$ (mm)</th>
<th>$V_{m}^{max}$ (mm/s)</th>
<th>Acceleration Tuning</th>
<th>Acceleration = 600 mm/s²</th>
<th>Improvement (%)</th>
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### REFERENCES


