

## Closure to "Discharge Coefficient Analysis for Triangular Sharp-Crested Weirs Using Low-Speed Photographic Technique" by C. Bautista-Capetillo, O. Robles, H. JÚnez-Ferreira, and E. Playán

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In the original technical note, an equation was proposed to determine the discharge coefficient ( $C_d$ ) of triangular sharp-crested weirs. The equation was based on the free-vortex theory. Supporting data were obtained using four fully aerated weirs. An adaptation of the low-speed photographic technique proposed by Salvador et al. (2009) was implemented to obtain the upper and lower nappe profiles.

The discussers A. R. Vatankhah and P. Peysokhan presented an in-depth analysis of our technical note. Additionally, they provided an alternative procedure to estimate  $C_d$ , leading to a comparison of the approaches used in both works. The discussers raised several issues that made part of our original research but could not be presented in the technical note due to space limitations. Now that the discussion is open, we are addressing these issues by introducing some of our findings for the particular case of the studied triangular sharp-crested weirs.

As the discussers pointed out, the free-vortex theory applied to sharp-crested weirs includes the analysis of the nonconcentricity of the streamlines, introducing a correction coefficient ( $k$ ). This consideration addresses the effects of viscosity, capillarity, surface tension and velocity distribution in the approach section, as well as streamline curvature due to the contraction of the liquid vein (Aydin et al. 2011; El-Hady 2011; Bagheri and Heidarpour 2010). The original paper proposed the following expression to determine  $C_d$ :

$$C_d = 3.750 Z_1 [k Z_0 + Z_1 \ln(Z_1 Z_1 + k Z_0)]$$

(1) where  $Z_0 = 0.682 [\tan(\theta_2)]^{0.044}$

(2)  $Z_1 = 0.445 [\tan(\theta_2)] - 0.098$

(3)  $k = 1.206 [h \tan(\theta_2)] - 0.014$

(4)  $Z_0$  and  $Z_1$  correspond to the  $Y/h$  and  $R_b/h$  ratios, respectively. Fig. 1 presents the experimental data supporting the development of Eqs. (2) and (3). In this Figure,  $Y$  is the flow depth at the maximum elevation section of the lower nappe;  $R_b$  is the radius of the streamline curvature at the lower nappe of the profile in segment OB; and  $h$  is the upstream depth (see Fig. 1 in the original paper).

For each tested flow, the experimental nonconcentricity of the streamlines ( $k$ ) was characterized using the low-speed photographic technique proposed by Salvador et al. (2009). The experimental values of  $k$  were plotted against the product of  $h$  and the vertex angle, expressed as  $\tan(\theta/2)$  (Fig. 2); the regression curve resulted in Eq. (4).

Furthermore, the discussers referred to predictive equations of the discharge coefficient presented by other authors. In Fig. 3 of the original paper, Eq. (1) was compared with some alternative equations to estimate  $C_d$  proposed in the literature. These equations and their limitations are presented in Table 1 to facilitate comparison with the equation we proposed.

Finally, the equation proposed by the discussers can be useful to estimate discharge in triangular sharp-crested weirs in the absence of the experimental data used in this research to derive the proposed Eq. (1).

## References

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Fig. 1.  $R_b/h$  and  $Y/h$  versus  $\tan(\theta/2)$  using upper and lower nappe quadratic equations

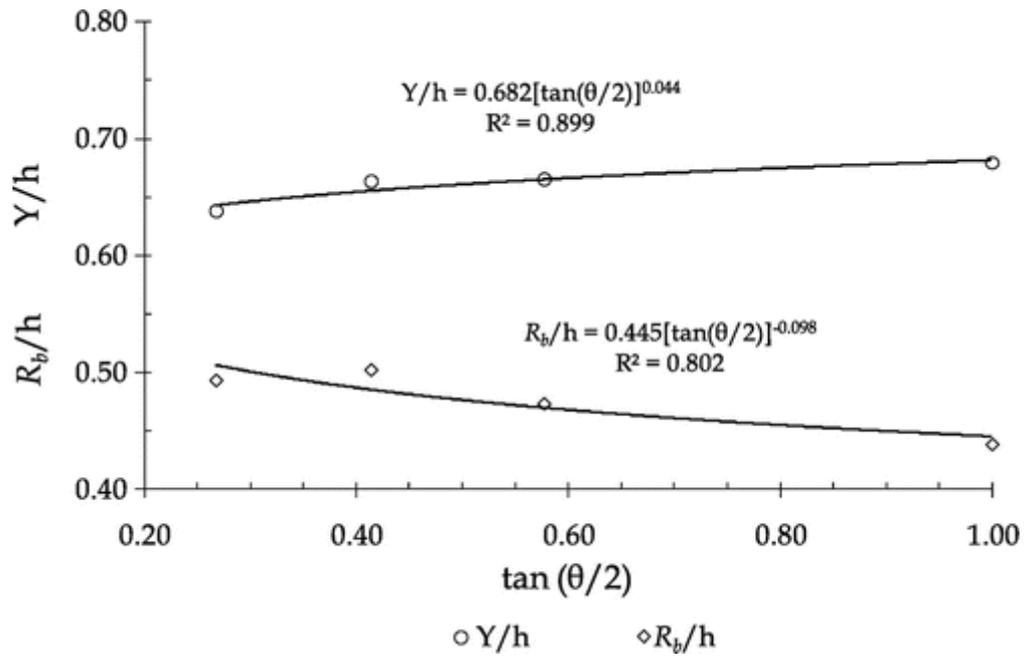


Fig. 2. Variation of  $k$  with  $h \tan(\theta/2)$

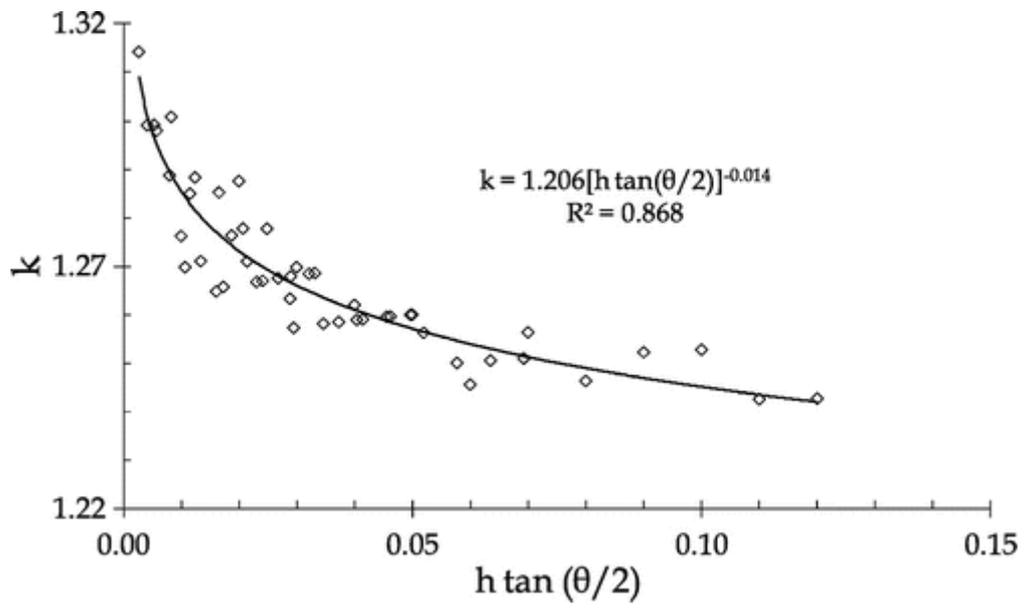


Table 1. Discharge Coefficient for Triangular Sharp-Crested Weir Proposed by Some Authors

<b>Author</b>	<b>Equation</b>	<b>Limitations</b>
Barr and Strickland ( <a href="#">1910</a> )	$Cd=0.566+0.0157h\sqrt{}$	For 90° V-notch
Cone ( <a href="#">1916</a> )	$Cd=0.576hj+0.00584Shj$	For 28–120° V-notch $j=0.0195S^{0.75}$ <i>S</i> is the slope of the sides of notch
Greve ( <a href="#">1932</a> )	$Cd=0.585[\tan(\theta/2)]^{0.004h^{0.03}}$	For 20–120° V-notch
Lenz ( <a href="#">1943</a> )	$Cd=0.560+Nha$	For 10–90° V-notch; <i>N</i> and <i>a</i> are functions of angle vertex weir
King ( <a href="#">1954</a> )	$Cd=0.595h^{0.01}$	For 60° V-notch
	$Cd=0.589h^{0.03}$	For 90° V-notch

Note: *Cd* = discharge coefficient. Equations adapted from Shen ([1981](#)).