Acoustic propagation trough internal gravity waves using normal-mode

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Abstract: In this paper we will interest ourselves on acoustic propagation through internal waves. This type of waves is generally due to variations of density on time scales of a few hours; they induce oscillations on the sound speed profile. We will discuss the effects of internal waves on acoustic propagation. We will use a one way-coupled mode approach to model our problem and present the results for two cases. In the first one, we will study the effects of the internal waves in a deep water problem. In the second one, we will present results for a shallow water case.

1. Introduction

Ocean sound speed presents variability in time of a few hours and in space of a few kilometers. This fluctuations are generated by meso-scale feature called internal waves. These are subsurface waves wich are propagating along interface separating two fluid layers of different density. They are considered to be a limiting factor in propagation of acoustic energy [5].

In this paper we will present a simulation of acoustic propagation through an internal wave field using normal modes. We will consider time as a constant in our problem, and two different normal modes solution: an adiabatic one and a coupled one.

2. Modelling ocean sound speed

Let *c(r,z,t)* be the ocean sound speed, a function of range *r*, depth *z* and time *t*. We will consider a range dependent sound speed profile which will consist of a deterministic range independent model plus a range dependent perturbation due to internal waves:

$$c(r,z,t) = \bar{c}(z) + \delta c(r,z,t) \qquad (1)$$

The independent sound speed profile is calculated using a Munk profile, typical for ocean depth in the range of 4000-5000m:

$$\bar{c}(z) = c_0\left[1 + \varepsilon\left(e^{-2(z-z_s)/z_s} - 2(z-z_s)/z_s\right)\right] \qquad (2)$$

Where z_a corresponds to the sound channel axis depth, ε is a constant fixed to *0.0737*, and $c_0=1500$ m/s. Most oceans in the world present stratified behaviour and their density gradient is in the form [1]:

$$N(z) = \left(\frac{-g}{\bar{\rho}}\frac{\partial \rho_p}{\partial z}\right) \qquad (3)$$

Where *N(z)* is called the local stability (or more commonly the Brunt-Vaïsälä frequency, or buoyancy frequency), $\bar{\rho}$ is the mean density in the water column, ρ_p the potential density and *g* the gravity constant. We followed Garret and Munk, and assumed a stratified ocean of the form:

$$N(z) = N_0 e^{-z/z_a} \qquad (4)$$

We note *w(x,y,z,t)* as the vertical displacement. It can be shown that *w* satisfies [2]:

$$\frac{\partial^2\left(\nabla^2 w\right)}{\partial t^2} + N^2(z)\nabla_h^2 w = 0 \qquad (5)$$

Now we will assume a solution of (5) of the form [2]:

$$w = W(j,k,z)e^{i(k_1x+k_2y-\omega(j,k)t)} \qquad (6)$$

Where *j* is the number of the mode, ω the frequency of the internal wave, *k* the internal wave wavenumber. Combining (6) and (5) we lead to:

$$\frac{\partial^2 W}{\partial z^2} + \left[\frac{N^2(z)-\omega^2}{\omega^2-\omega_i^2}\right]k^2 W = 0 \qquad (7)$$

Where ω_i is the inertial frequency. We will use the value $\omega_i=1$ cph in this paper. The boundary conditions are set to zero at the top and the bottom of the ocean. We will use the following normalization for *W(j,z)*:

$$\int_0^H (N^2(z)-\omega^2)W(j,z)W(j',z)dz = \delta_{ij} \qquad (8)$$

Hence, each frequency ω has its own set of modes. Therefore the vertical displacement can be expressed as the sum over all the modes and over frequency [3]:

$$\xi(r,t) = \int_{\omega_i}^{N}\sum_j A(\omega,j)W_{\omega,j}(z)e^{i(kr-\omega(j,k)t)} \qquad (9)$$

Where $A(\omega,j)$ is the modal amplitude associated with the Garret-Munk spectrum and the jth mode, and H the bottom depth. $A(\omega,j)$ is a gaussian random variable [4]:

$$A(\omega, j) = \left[2rB(\omega)H(j)\int N(z)dz)\right]^{1/2} \qquad (10)$$

Where B(w) and H(j) are of the form:

$$\begin{cases} B(\omega) = \dfrac{2\omega_i}{\pi\omega\sqrt{\omega^2 - \omega_i^2}} \\ H(j) = \dfrac{\left(j^2 + j_*^2\right)^{-p/2}}{\sum (j^2 + j_*^2)^{-p/2}} \end{cases} \qquad (11)$$

Where p and j_* are modal parameters.

To solve (7) we use a three point finite difference scheme, and a QR algorithm to get the eigenvectors and eigenvalues of our problem.

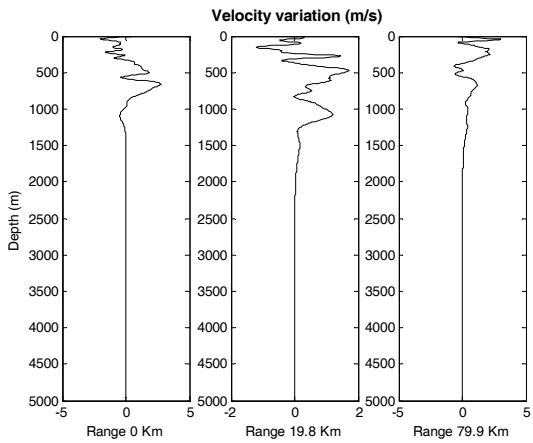

Figure 1. Velocity variation as a function of depth at different ranges, for an internal wave of 2 cph.

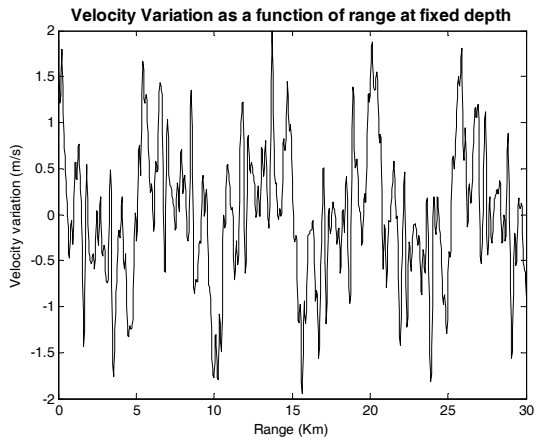

Figure 2. Velocity variation at a fixed depth, for an internal wave of 2 cph.

3. Normal mode solution

3.1. Range-independent waveguide

The main idea of the normal mode solution is to solve the wave equation using a variable separation method [6]. This modes are comporting like a set of vibrating strings. The total field is computed summing the modes, each one weighted in accordance to the source depth.

The first step is to get two separate equations of the wave equation. For this we write the pressure as:

$$P(r, z) = \phi(r)\psi(z) \qquad (12)$$

Thus we get two separate equations:

$$\begin{cases} \psi''(z) + \left[\omega^2 / c^2(z) - k_{rn}{}^2\right]\psi(z) = 0 & \text{(a)} \\ \phi'' + \dfrac{1}{r}\phi' + \zeta^2\phi = 0 & \text{(b)} \end{cases} \qquad (13)$$

The solution of (13.b) is a Hankel function of zero-th order and of the first kind. The problem is now to solve (13.a) known as Helmholtz equation. In isovelocity problems, there exist exact solutions of (13.a), but in the more general case, like velocity depth dependent one, there is no analitycal solution available. Thus we fall back on numerical solutions. We use the algorithm proposed by Porter & Reiss [7], to get the following linear system:

$$A = \begin{bmatrix} a_1 - h^2k_r^2 & 1 & 0 & . & . & . & 0 \\ 1 & a_2 - h^2k_r^2 & 1 & 0 & . & . & 0 \\ 0 & 1 & a_3 - h^2k_r^2 & 1 & 0 & . & 0 \\ . & . & . & . & . & . & . \\ . & & . & . & . & . & 0 \\ . & & & . & 1 & a_{N-1} - h^2k_r^2 & 1 \\ 0 & . & . & . & 0 & 2 & a_N - h^2k_r^2 - 2hf(k_r^2)/g(k_r^2) \end{bmatrix}$$

Where:

$$a_i \equiv -2 + h^2\omega^2 / c^2(z_i),\ i=1,2,..,N_1 \qquad (14)$$

And h is the depth increment, k_{rn} the modal wave number, ω the source frequency, and $c(z)$ the ocean velocity. The boundary conditions $\psi(0)= \psi(H)=0$ implie that f and g are set to zero.

The final solution is obtained by summing modes obtained using (14):

$$P(r, z) \approx \frac{i}{4\rho(z_s)} \sum_{n=1}^{M} \psi_n(z_s)\psi_n(z)H_0^1(k_n r) \qquad (15)$$

Where $\rho(z_s)$ is the density at the source depth z_s.

3.2. Range-dependent waveguide

We follow Evan's formulation for range dependent waveguide [8]. We divide the axisymetric problem in range independent waveguide, and we seek a solution of the form for the j-th segment:

$$p_j(r,z) = \sum_{m=1}^{M} \left[a_m^j \hat{H}1_m^j + b_m^j \hat{H}2_m^j \right] \psi_m^j(z) \qquad (16)$$

Where:

$$\begin{cases} H1_m^j(r) \approx \sqrt{\dfrac{r_{j-1}}{r}} e^{ik_{rm}^j (r - r_{j-1})} \\ H2_m^j(r) \approx \sqrt{\dfrac{r_{j-1}}{r}} e^{-ik_{rm}^j (r - r_{j-1})} \end{cases} \qquad (17)$$

Imposing contuity of pressure and matching radial particle velocity, we get the following relationship between the j-th and (j+1)-th segment:

$$\begin{bmatrix} a^{j+1} \\ b^{j+1} \end{bmatrix} = \begin{bmatrix} R_1^j & R_2^j \\ R_3^j & R_4^j \end{bmatrix} \begin{bmatrix} a^j \\ b^j \end{bmatrix} \qquad (18)$$

Where R_i^j are the coupling matrix between modes. For convenience, we will neglect backscattering. Therefore, we get a simplified single-scaterred recursion [6]:

$$a^{j+1} = R_1 a^j \qquad (19)$$

One may also consider an adiabatic approximation by neglecting coupling between modes. Hence the total pressure field is [6]:

$$p(r,z) = \frac{i}{\rho(z_s)} e^{-i\pi/4} \sum_{m=1}^{\infty} \psi_m(z_s) \psi_m(z) \frac{e^{i\int_0^r k_{rm}(r')dr'}}{\sqrt{\int_0^r k_{rm}(r')dr'}} \qquad (20)$$

4. Results

4.1. Deep water problem

Now we will consider a deep ocean waveguide with a Munk deterministic velocity profile. We calculate the velocity perturbation for a 1 cph internal gravity wave and the buoyancy profile define in (4). Ocean depth is 5000m. We will studie propagation of a monochromatic source at 3 different frequencies, and compute the pressure field for different number of modes.

In figure 6 we present the coupling between modes for the different frequencies mentioned above. Clearly we see that the coupling at 25 Hz is lesser than at 100 Hz and 50 Hz, as expected. Thus at 25 Hz, the propagation seems to be more adiabatic. But it can't be refered as adiabatic, since there is coupling between the first 30 modes. For modes of order above 30, we can neglect the coupling (<1%). We see from figure 4 and figure 5 that there is a clearly coupling between modes wich affects the acoustic propagation.

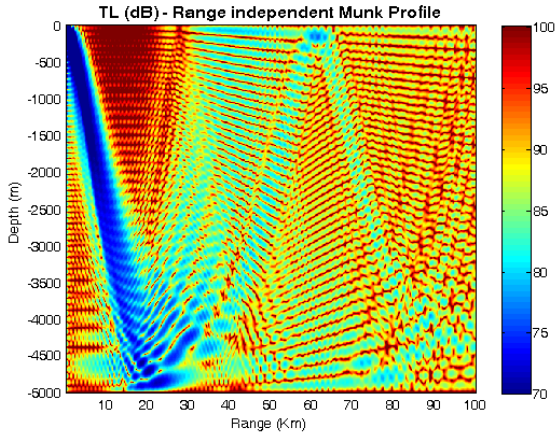

Figure 3. Transmission Loss for a 25 Hz and the Munk Profile of equation (2).

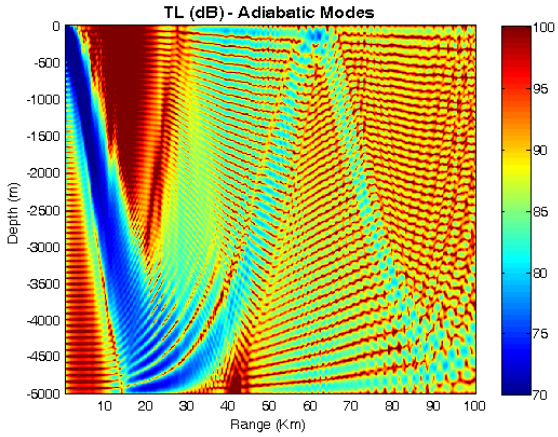

Figure 4. Transmission Loss for a 25 Hz source, using adiabatic modes.

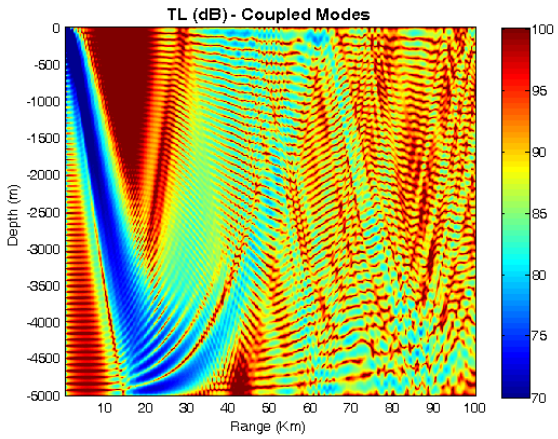

Figure 5. Transmission Loss for a 25 Hz source, using coupled modes.

We can see that the adiabatic solution matchs closely the range-independant solution (munk profile). But we see from figure 5 that we can't neglect the coupling between modes, since the presence of the internal wavefield clearly affects the propagation of the wave through phase changes.

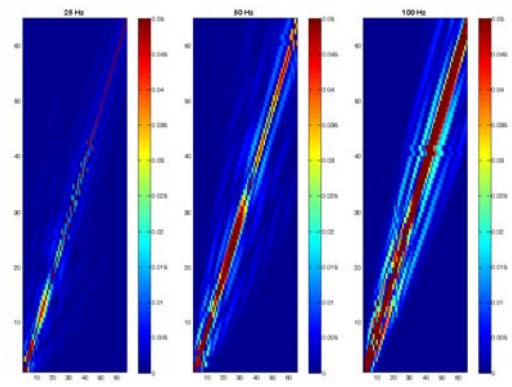

Figure 6. Coupling between modes for 3 different frequencies.

4.2. Shallow water case

Now we will consider a shallow water case problem. We use the buoyancy profile of figure 7, a constant density layer of 1g/cm^3, and an internal wave frequency of 1 cph.

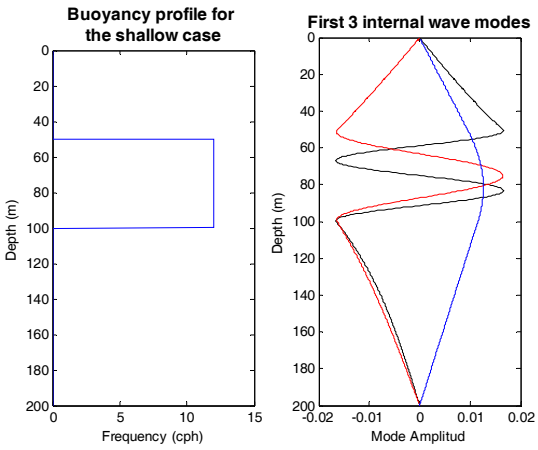

Figure 7. Buoyancy profile in a shallow water case. From left to the right: 25 Hz, 50 Hz, and 100Hz.

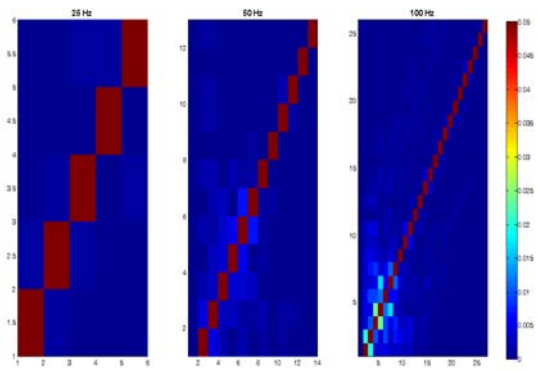

Figure 8. Modes coupling for the shallow water case. From left to the right: 25 Hz, 50 Hz, and 100Hz.

We can see from figure 8 that the propagation is totaly adiabatic at 25 Hz, and that modes coupling increase with frequency.

5. Conclusions

From this sudy, we have shown how the internal wave field will affect the acoustics propagation through coupling between modes: we can note that it affects the lowest modes first, and that coupling between modes increases with frequency.

It seems that the propagation in shallow water is more adiabatic than in the deep water one. We can explain this if we consider that there are more modes excited in a deep water problem, so that there will be more coupling between them.

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