Semiconductor laser dynamics under polarized rotated optical delay feedback and frequency detuning

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“If you can’t fly then run, if you can’t run then walk,
if you can’t walk then crawl, but whatever you do
you have to keep moving forward.”

Martin Luther King Jr.
Abstract

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This Master Thesis numerically studies the changes in the dynamics of a semiconductor laser with delayed feedback considering a system of two modes (TE and TM). We restrict ourself to the orthogonal feedback case -i.e. when TE mode feeds the TM mode and the TM mode feeds the TE mode, both with delay- when a detuning exists between modes. The Lang-Kobayashi model is used to describe the laser dynamics and to understand the underlying effects. Particular emphasis is put to the detuning parameter between modes. The inclusion of a detuning accounts for an effect present in practical situations that may originate changes in the dynamics in an unexpected way. The objective is to extract information and conclusions about the changes induced by the feedback to the system in a qualitative way.
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Chapter 1

Overview

1.1 Outline

The present thesis is structured in the following way. In Chapter 1 we introduce the topic of lasers, including why they are relevant, what their are components and briefly describe the different kinds of semiconductor lasers. Later on we explain the concept of delayed feedback, and the specific feedback considered here, polarized rotated optical feedback. At the end we explain the detuning effect.

Chapter 2 explains the model used for the simulations. We describe the physical meaning of the model and of the different contributions. We start from a single laser and include later the feedback effects. Finally we introduce the detuning and the parameters values completing the model.

Chapter 3 presents simulation results in three blocks. The first block compares the results obtained in the presence and absence of detuning in the system. The second block includes results for different parameter values, including coupling strength, differential gains and detuning. The third block presents particular results with fixed detuning and differential gain for different coupling strength values.

Chapter 4 contains a summary and an overview of the conclusions obtained throughout the work and possible extensions of research in the topic.
1.2 Introduction

In the present report we study the dynamics of semiconductor lasers (SL) in the presence of polarization rotated optical delayed feedback when there is a detuning between the two polarization modes. Lasers are very interesting devices for their special properties and extended use, far beyond any initial expectations when invented in 1960 [1]. Even nowadays we are not fully aware of how often we turn on a laser every day. They accomplish many things that other devices can not do. One of the most used laser types is the semiconductor laser [2]. Their non-linearities make them an interesting non-trivial system to study. In some cases these non-linearities can even be beneficial. An enormous amount of papers and several books have been published on semiconductor lasers [3–5]. One of the most intriguing aspects is their sensitivity to delay optical feedback. The delayed feedback is an effect that was observed in the seventies [6–9]. Fraction of the out-coming light of the laser is reintroduced into the laser, perturbing the usual steady dynamics. Understanding this situation and consequences was, and still is, a matter of study. There can be many different situations and possibilities of the system configuration when mentioning feedback, from lasers with just one mode, different polarization directions, etc [10]. In this thesis we consider a semiconductor laser with two modes under orthogonal polarization and cross-correlation feedback. We explicitly consider the detuning (frequency difference) between these two modes.

1.2.1 Importance of lasers in nowadays society

Lasers have many unique characteristics that make them the best choice for many uses. They have light efficiency higher than other light sources as, e.g., light bulbs. A Yb:YAG solid state laser can have an efficiency around 30% [11], while a 40W tungsten incandescent light bulb can have an efficiency around 2% [12, 13]. Together with the wide range of possible output powers, lasers emerge as a tool for transforming electric energy into light, from just some milliwatts up to $1.5 \times 10^5$ J in a 10 ns pulse [14]. Directionality of the emitted ray allows to use light efficiently, and precisely in the spot where you want it. Power and directionality allow using lasers when is necessary to place energy locally, to heat, burn or cut. In industrial manufacturing, lasers are used to engrave serial numbers on surfaces or to remove superficial layers, slightly burn the surface of wood (carbonization), or just to cut through metal plates 1 cm thick [15]. Lasers spectral linewidth can be wide as $\sim MHz$, and down to $\sim KHz$ [11, 16]. This chromaticism is unimaginable in other photonic devices. This allows lasers to be useful in a wide number of LIDAR techniques (LIght Detection and Ranging), and spectroscopy. Medical science has taken use of dye lasers for many applications thanks to the frequency emission of these lasers and the absorption factor of living tissues [17]. But the widest use of lasers is
found in telecommunications, used as emitters of information in the form of light. The light is transmitted using optical fibers even through the bottom of the oceans.

The narrow bandwidth is related to another important characteristic of laser light, which is the inherent high coherence both in time and space. This may be the most important difference between lasers and other light sources. It is said an electric field present coherence when there is a fixed phase relationship at different times and positions (referring to temporal and spatial coherence respectively). This characteristics allows for precise interferometric techniques and for the construction of optical accelerometers using Sagnac effect [11].

Finally it is also worth to highlight the economical importance of lasers in international markets. The laser industry grew from its beginning in a exponential fashion over 30 years. The growing rate has only been interrupted by two economical crisis, the dot – com crisis in the 2000 and by the Subprime mortgage crisis in 2007. The effect of these crisis affected in different ways on the lasers markets. The first one affected the telecommunications market sector once the bubble burst. The realisation of the investor that these companies were not as profitable as they seemed at the beginning just because they had .com or e— in their names triggered on March 2000 a fall on NASDAQ Composite. Once the bubble burst and companies closed, fibre optics installations dropped, so that laser market sector fell that burst then. Before this crisis, laser market was worth around 7B$, and the dot – com crisis reduced it to around 4B$ in 2 years. The market recovered its position just in time to suffer the 2007 Subprime mortgage crisis. This crisis affected (and still does) investments, research, and development in companies of very different kind. And since lasers are present in many economical and production sectors, the lasers revenue was affected in every sector. This time loses were not as big as previously. Lasers revenues fell from 7B$ to 5B$, but this time recovery was much more quickly. Nowadays, laser market revenues are worth close to 9B$. The different lasers sectors are not equally big. More than half of the cake are represent by the the communications and the material processing sectors (31% and 25% respectively). Optical storage (14%) follow the first two. The remaining 30% is divided among lithography, military R&D, medical applications et al. An important fact here is that laser market has grown exponentially from its beginning, and drops were not cause because lasers were overpowered by other devices or processes were lasers are not involved. The drops were indirectly caused by external reasons. Lasers have proven excellent in their applications, source of rapid grow of the market after the crisis. Still today market does not present any clue of stillness. True is that recently growth has not been as explosive as other years (remains of this last crisis), but many companies are optimistic not only about the wide future growth of the laser market, but also of the new application fields lasers will open in the new years thanks to investment in research and development involving lasers [2].
1.2.2 Ingredients for a laser

To describe the laser we should explain first the origin of the word laser. Laser is an acronym from the words “light amplification by stimulated emission of radiation”. First person who think about the core mechanism of lasers was Albert Einstein when in 1917 introduced the concept of stimulated emission. This concept contemplates the possibility where a photon interacts with an excited atom (or molecule). From this interaction the atom (or molecule) returns to the ground state emitting a photon which has the same frequency, phase, direction and polarization than the original. Before the laser there was the maser (another acronym referred to “microwave amplification by stimulated emission of radiation”). It was invented in 1954 by two independent groups; Nicolay Basov and Alexsandr Prokhorov in U.R.R.S. at the Lebedev Institute in Moscow, and by Charles Townes and Jim Gordon in the U.S.A. at the Columbia University in New York. Using ammonia gas (a 2-level system), soon after that, Townes and Gordon kept working on the idea of creating masers of visible light. In 1958 they introduced the maser in between two highly reflecting mirrors. This turned the maser into a laser (or optical maser as called then by its creators). This device needed to be improved, and many research facilities worked to create the first working laser. At Hughes Lab in the May of 1960, one man realized that high gain pulsed oscillations could be obtained in ruby, optically pumped by a flash lamp. This man was Theodore Mainman. Since then, lasers’ field has exploded leading to many different kinds of lasers using different materials and techniques [18, 19].

But all lasers share three basic ingredients. The ingredients are a pump source, a gain medium, and an optical resonator. We can see an illustration of these elements conforming a laser in Figure 1.1.

The pump is a form of external energy introduced into the system. It is required to excite the population (atoms or molecules) and create population inversion (where there are more atoms or molecules in the excited state than in ground state). The most common are optical and electrical pumping. Optical pumping means that the energy is introduced using light exploiting...
the broadband absorption lines of the gain medium to introduce efficiently energy in the laser. This light can be from an incoherent or a coherent source. As incoherent source, flash lamps or high power ones are suited as pump sources. They are filled with noble gases at medium or high pressures. $Ar^+$ lasers, Nd lasers of second or third harmonic generators, and even semiconductor lasers are used as coherent sources. Electrical pumping is usually more convenient. It involves simply the application an electrical flow through the gain medium. It is particularly useful when absorption lines are thin, for example in gas lasers. In semiconductors lasers the pumping is mostly done electrically (some VCSELs, Vertical Cavity Surface Emitting Laser, are pumped optically) [20].

The gain medium provides the excited population so photons will generate clone photons of themselves leading to coherent amplified emission. This is obtained thanks to the stimulated emission process. As a chain reaction, these two photons will stimulate the emission of two other photons, and so on, until there are no population in the excited state. The gain medium can be very different from one laser to another, and this difference leads to the classification of lasers by material. Some of them are solid state lasers (like Nd:YAG or ruby lasers), gas lasers (like He-Ne or $CO_2$) and dye lasers (a liquid organic dye is the gain medium, like DTTC or BBQ). But the ones we study here are semiconductor lasers. The gain medium is a p-n junction. The current from the electric pump crosses the junction as electrons in the conduction band and holes in the valence band. These pairs of electron-holes are the carriers, and the average time they take to spontaneously recombine is called carrier lifetime. Near the junction is the depletion zone, where is more likely that electron and holes meet, recombine and emit from this encounter a photon. This photon will have an energy equals to the energy difference between the conductance and the valence bands [20].

The last ingredient is the optical resonator. This conforms a cavity where the light is maintained; and keeping some light inside is necessary to achieve a laser action. Having some light inside the cavity makes the coherent stimulated emission process more efficient. Even without the resonator the light going through the medium would leave the system amplified by stimulated emission. However, the process would not be sustained. The optical resonator keeps certain amount of light inside the medium. Doing this, this light is amplified many orders of magnitude compared with the spontaneous emission. The result will be a coherent light. The resonators can be very different. It can be formed by two mirrors with reflectivities $R_1$ and $R_2$, planar or curved mirrors whose radii depends on the length of the medium. It can be a fiber ring, a micro pillar with distributed Bragg reflectors, or even a photonic crystal. In semiconductor lasers the most usual situation is with two planar mirrors created by the difference between refractive indeces inside and outside the laser. Due to this characteristic semiconductor lasers have mirrors with lower reflectivity compared with other lasers. They store less light intensity than other lasers. This makes semiconductor lasers “open”, while others are “closed”. Comprehensible since these “open” lasers emit about 70% of the light generated, while “closed” lasers emit
Figure 1.2: Illustration of an edge-emitter laser. It presents the p-n junction and light beam shape as exits the medium.

around 0.01%. This difference makes also semiconductor lasers more susceptible to the external light, perturbing the system in undesired ways. But also this effect can be of interest and will be referred to in following sections [20].

**Polarization of the light**

The polarization of the light refers to the orientation of the electric field as it travels in space. The polarization state is made of orthogonal components of the electric field, which amplitudes and phases determine the final polarization state. If the electric field keeps the polarization, light is linearly polarized. If on the contrary polarization changes, light has elliptical polarization. [21]

In semiconductor lasers the polarization of the light is mainly determined by the structure of the junction. Edge-emitting lasers (EEL) have a planar gain medium, with dimensions or length \( \sim 200\mu m \), width \( \sim 20\mu m \), and depth of \( \sim 5\mu m \). Light travels along the longest dimension because it interacts more with the gain medium. Here the gain medium is the active layer, a zone in the double heterostructure of the junction between the p-doped and the n-doped zones, which keeps electron and holes much closer and confined for an efficient recombination. The outcome light of EELs is linearly polarized. The rectangular shape of the active layer makes an astigmatic light beam with an elliptical shape due to diffraction. The ellipse is shown in Fig. 1.2, where the longest axis is in the direction of the depth of the layer. But the amplitude of the fields in each direction differs. The component parallel to the layer is the Transverse Electrical (TE). The component with large diffraction is the Transverse Magnetic (TM). In EEL lasers usually the TE mode is more energetic than the TM mode.

VCSELs (Vertical Cavity Surface Emitting Lasers) are illustrated in Fig. 1.3. These lasers emit light perpendicular to the gain medium plane. Due to the small thickness of the layer the the
gain per round trip is low, but facet reflectivity of VCSELs are much higher than in EELs. This high reflectivity is achieved with Distributed Bragg Reflectors (DBR). These devices have a gain medium with cylindrical symmetry, which makes all polarization directions possible at the output. They can emit linear polarized light, but polarization switching easily occurs. The diffraction of the beam is the same on all directions, unlike EELs.

1.2.3 Polarization rotated optical delayed feedback

We consider delayed optical feedback as a portion of light that is emitted by the laser and reinjected into it a time later. This is an important effect since even small amounts of feedback can perturb the dynamics of the SL and the spectral characteristics significantly. This feedback can be induced for instance by placing a mirror at a certain distance form the laser. Optical feedback can appear in applications like optical recording and optical communications. Back reflections from optical fibers, lenses, or surfaces can hit the laser and perturb its state. The fact that semiconductor lasers are “open” makes this optical light to enter more easily. Due to this feedback different dynamical regimes can develope. One of the regimes is Low Frequency Fluctuations (LFF) regime, occurring for injection currents close to threshold [22]. These instabilities are present in the light output, characterized by peaks of increasing power followed by drop outs and starting over with low peaks. Important parameters related to optical feedback are the optical length of the external cavity, the ratio of injected and emitted light, and the phase accumulated by the field in the external cavity.

The polarization rotated optical feedback (PROF) refers to the idea of feeding back light into the perpendicular mode; the laser emits in two modes that re-inject one into the other. This
is, e.g., done by taking the TE mode, rotate its polarization by $\pi/2$, and inject it back into the laser; and the same for the TM mode, turning it into TE’s polarization direction. This creates a cross feedback between modes.

### 1.2.4 Frequency detuning

The incorporation of the detuning has its origin in the fact that the modes oscillate at a certain frequency. These frequencies may be not the same; may be separated by some MHz or even GHz. It is known that the operation of feeding back the modes to the laser changes abruptly the dynamics of the laser, but what would happen if this is done at a different frequency of the mode used to fed back. We define the detuning as the difference between the frequency of the two modes.

The origin of this difference can arise in several forms. Light with different propagation orientations (TE/TM) may see slightly different refraction indices, maybe due to strain and thus oscillate at different frequencies. The semiconductor photonic devices are susceptible to wavelength emission change due to many factors, temperature and injection, for example [20]. In addition, one can use non-linear optical devices to shift the frequency of the light so it is detuned by a certain amount. The use of the detuning in this fashion has been studied in frequency-shifted feedback lasers [23–25].

In this work we consider that the two polarization modes do not emit at the same frequency and, without loss of generality, we introduce the detuning in the TM mode. In the last section of Ch.2 we show how we introduce this new feature in the model.
Chapter 2

Modelling of semiconductor lasers, feedback, and detuning

2.1 Rate equations for a semiconductor laser.

The rate equations to describe the dynamics of a semiconductor laser originate from the Maxwell-Bloch equations (MB). The MB are obtained with a model through a semi-classical treatment of the light interacting with the matter (the electromagnetic waves are treated classically, while matter is treated in a quantum way). The MB equations is a set of three equations, involving the temporal evolution of the electric field, the polarization of the medium, and the carriers number. Using a slowly varying envelope approximation and an adiabatic approximation one can disregard the polarization of the medium. In this way we end up with just two equations. These equations (eqs.2.1, 2.2) are suitable for almost all semiconductor lasers, and are the one used to described a single longitudinal mode laser. The adiabatic approximation considers that the polarization of the medium changes very fast compared to the electric field and the carriers number. This means that, in time scale of the electric field and the carriers number the polarization has already decayed to its steady state value. The result is a set of two equations describing the dynamics of the semiconductor laser.

From the approximations done to the MB equations, the lasers can be classified in three groups: class A, class B, and class C. Class C lasers require the consideration of the three equations to describe their behaviour. Three equations may turn things more difficult, but in fact they allow to experience chaos (in fact, lasers were the first experimental system where chaos was observed). In contrast Class A lasers can be modelled with a single non-linear field equation. Class B lasers, such as most semiconductor lasers, require the electric field and the carriers number evolution equations to be described. The equations for a typical semiconductor laser read [26]:

9
\begin{align*}
\frac{d E(t)}{dt} &= \frac{1}{2} (1 + i\alpha)(G(n(t) - n_0) - \gamma_{ph})E(t) \\
\frac{d n(t)}{dt} &= J - \gamma_s n(t) - G(n(t) - n_0)|E|^2
\end{align*}

Equation 2.1 describes the time evolution of the electric field. RHS starts with $1 + i\alpha$, where $\alpha$ is the linewidth enhancement factor, a term proportional to the rate of change of the real part of the susceptibility with the carrier density and inversely proportional to the rate of change of the imaginary part of the susceptibility also with the carrier density. Term $G(n(t) - n_0)$ is related to the stimulated emission. The differential gain coefficient ($G$) appears multiplying the amount of carrier density ($n(t)$) minus the carrier density at transparency ($n_0$). $\gamma_{ph}$ is the inverse of the expected photon lifetime (this mechanism reduces the number of photons in time). All multiplies the instantaneous electric field. Differential gain is considered here as constant. The gain could be expressed by a non-linear term of the form $G = G_0 \frac{1}{1 + \epsilon |E|^2}$, where $\epsilon$ is the non-linear saturation coefficient (defined as the power value for which gain is reduced to 1/2 of the original gain). In this work we do not consider the saturation gain because all treatments and simulations are done for low values of the injection current, where the differential gain can be considered a constant.

Eq. (2.2) describes the time evolution of the carrier density. Term $J$ represents the injection current of carriers in the system. Injection of carriers comes from the electric injection of electron-holes pairs ($I$). When this current is divided by the cavity volume ($V$) is the current density, but when it is divided by the electron charge ($e$), we have the injected carrier density: $J = \frac{I}{Ve}$. The next term is a relaxation term, from the natural decay through non-radiative recombination of carrier density in the system or spontaneous emission. This term is proportional to the carrier density and to the inverse of the expected lifetime of the carrier density, $\gamma_s$. The last term is related to the carrier density reduction due to the stimulated emission. This term is proportionally to light intensity ($|E|^2$).

We can do a stability analysis on this set of equations. From it we can obtain the steady states and its stability depending on the injection current ($J$). The steady states are calculated setting variables time evolution to zero. The steady value have subscript $st$.

\begin{align*}
0 &= \frac{1}{2} (1 + i\alpha)(G(n_{st} - n_0) - \gamma_{ph})E_{st} \\
0 &= J - \gamma_s n_{st} - G(n_{st} - n_0)|E_{st}|^2
\end{align*}

Isolation of the variables $E_{st}$ and $n_{st}$ presents 2 steady states. For simplicity we take $E(t) = \sqrt{I(t)}e^{i\phi(t)}$ where $I = |E|^2$ is the light intensity and $\phi$ the optical phase. First steady state:
Chapter 2. Modelling of semiconductor lasers, feedback, and detuning

\[ n_{st,1} = \frac{J}{\gamma_s} \]
\[ I_{st,1} = 0 \]

Steady state 1 has no light intensity while the carriers density grows linearly with the injection current. Second steady point:

\[ n_{st,2} = n_0 + \frac{\gamma_p}{\gamma_s} \]
\[ I_{st,2} = \frac{1}{\gamma_{ph}} (J - \gamma_s (n_0 + \frac{\gamma_p}{\gamma_s})) \]

Steady state 2 presents a constant value of carriers density. Light intensity grows linearly with the injection current. Perturbation theory of the model equations tells us the stability of the steady states. The model presents a transcritical bifurcation when injection current \( J \) changes. \( J \) range is for positive values. The bifurcation appears for a certain value of current called threshold current \( J_{th} = \gamma_s (n_0 + \gamma_{ph}/G) \). Below \( J_{th} \) steady state 1 is stable and steady state 2 is unstable. Above \( J_{th} \) steady state 1 is unstable and steady state 2 is stable. If the system starts outside the unstable steady state, it will evolve towards the stable steady state. This means the system will evolve towards steady state 1 if \( J < J_{th} \), and therefore the laser is not stimulated. On the other hand, if \( J > J_{th} \) the system will evolve towards steady state 2 and the laser will emit stimulated light. The stability analysis shows also that the steady state 2 is a stable focus state. Evolution of the system towards or from a focus states is through oscillations. In this model, the frequency of these oscillations define the relaxation oscillation frequency. Power and \( n_{st} \) are shown in Fig. 2.1 for values of \( J \) in the range \([0, 2]\). Stationary power is related to steady light intensity \( I_{st} \) by Eq. 2.5 using speed of light \( c = 3 \times 10^8 \text{ m/s} \), dielectric constant \( (\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}) \), and beam cross section (using a typical value of \( A = 1.65 \times 10^{-8} \text{ m}^2 \)) [?].

\[
P(nW) = \frac{c \epsilon_0 A}{2} |E|^2 = 2.19 \times 10^{-20} |E (N/C)|^2 \tag{2.5}
\]

2.2 Optical Feedback

In this section we present the model used for the description of the optical feedback in the laser. As already mentioned, semiconductor lasers are sensitive to feedback from light reflected outside the device. This leads to undesired results in practice, but also to intriguing situations, affecting the dynamics of the semiconductor laser in a strong way. This effects were known, and early reports about them appeared in 1979 [27, 28]. The paper published by R. Lang and K. Kobayashi in 1980 [29] proposed a model to describe the optical feedback situation in a single longitudinal mode laser, experimentally and theoretically. The equations and their origin are explained in Sec. 2.2.1. The feedback is injected not instantaneously, but with a delay time. This delay time can be small or big depending on the characteristic time scale of the
solitary laser defined by the inverse of the relaxation oscillation frequency. In this work the delay time is chosen to be long, therefore we consider long external cavity regime. The delay in the system increases the dimensionality of the system, theoretically, to infinite. In practice the dimensionality is not infinite, but still very high. Such high dimensional system is candidate to present different dynamical regimes such as chaos [22, 30]. This high dimensionality arises from the fact that it is necessary to know the initial conditions in a continuous time interval of length equal to the delay time. This kind of feedback is usually called coherent feedback. It can be used as source for the generation of random bits [31], chaotic transmission of information [32], and in chaotic LIDARs [33].

The Lang-Kobayashi model describes a situation were polarization of the light is maintained. In an external cavity configuration this is not necessarily true. If we think in external cavities build upon fibers, polarization of the light can change due to many factors such as torsion, stress, impurities, imperfections in the shape or the material, etc. Even powerful magnetic fields close to the fiber can change the polarization state of the light travelling inside (this is because of the Faraday effect, which we explain shortly below). Our considerations involve a model where the polarization of the light has changed through the external cavity by $90^\circ$. This means that modes are fed back with the light coming from the other mode, not by themselves. Recalling what was mentioned in Chapter 1, the rotation of the polarization of the light results in the TE mode being fed by light from TM mode, and the TM mode by the TE mode. This scheme is called Polarization Rotated Optical Feedback (PROF) with 2 mode dynamics.

In the present work we use a modified version of the Lang-Kobayashi approach to account for
Figure 2.2: Illustration of the semiconductor laser in an external cavity used to derive the feedback equations.

the PROF with 2 mode dynamics. In the following subsections we explain the Lang-Kobayashi model, and then, modify it to account for PROF with 2 mode dynamics.

2.2.1 Lang-Kobayashi equations

The Lang-Kobayashi equations account for certain approximations and considerations. They assume that the laser operates in a single longitudinal mode, the feedback is relatively small or moderate, and that the external cavity makes the delay time much longer than the round-trip time in the laser (usually of some $ps$). The configuration of such situation is depicted in Fig. 2.2. This new situation requires modification of the rate equations. The reinjected light does not affect the carrier density in the gain medium, therefore the time evolution of carrier density equation remains the same.

In the optical resonator of the laser we can think of the light travelling as forward and backward waves in the $x$ direction, $E^+(t)$ and $E^-(t)$ respectively. The mirrors have a certain amplitude reflectivity $r_i$ and power reflectivity $R_i = r_i^2$. The length of the external cavity can be expressed as a function of the delay time as $2L_{ext} = c\tau/\eta$, where $\tau$ is the delay time and $\eta$ is the refractive index of the external cavity. The contributions to $E^-(t)$ are two, the contribution from the reflected $E^+(t)$ and the contribution from multiple reflections of the field at $x = L$. These can be written as follows [29], [34], [35], [36]:

$$E^-(t) = r_2 E^+(t) + (1 - r_2^2)r_3 E^+(t - \tau)e^{-i\omega\tau} + (1 - r_2^2)r_3(-r_2r_3)E^+(t - 2\tau)e^{-i2\omega\tau} + \ldots$$

$$+ (1 - r_2^2)r_3(-r_2r_3)^2E^+(t - 3\tau)e^{-i3\omega\tau} + \ldots \quad (2.6)$$

The angular frequency of the mode is $\omega$. The external contribution ($E^-_{ext}(t)$) can be then written as:
\[ E_{\text{ext}}(t) = - \frac{(1 - r_3^2)}{r_2^2} \sum_{n=1}^{\infty} (-r_2r_3^n) e^{-i\omega \tau} E^+(t - n\tau) \] (2.7)

Considering weak feedback, only the first reflection matters. Then the sum can be approximated to its first term, relating delay for just one round trip.

\[ E_{\text{ext}}(t) \simeq - \frac{(1 - r_2^2)r_3^3}{r_2} e^{-i\omega \tau} E^+(t - \tau) \] (2.8)

This term, divided by \( \tau_L = \frac{L n_2}{c} \) (laser cavity round trip time), is the contribution to the rate equation of the field due to the delayed feedback. By adding it to the rate equation, Eq. (2.1), we obtain the well-known Lang-Kobayashi equation, Eq. (2.9).

\[ \frac{dE(t)}{dt} = \frac{1}{2} (1 + i\alpha) \left( G(n(t) - n_0) - \gamma_{ph} \right) E(t) + k_{\text{inj}} E^+(t - \tau) e^{-i\omega \tau} \] (2.9)

The feedback coupling parameter \( k_{\text{inj}} \) in the feedback term is defined using the parameters in 2.10. This term accounts for the coupling strength. The parameter \( \beta \) accounts for power losses introduced by the external cavity by diffraction, dispersion in the medium or any other sources, and has a value between 0 and 1. In what follow, we focus on changing the parameter \( k_{\text{inj}} \) instead of the parameters defining \( k_{\text{inj}} \).

\[ k_{\text{inj}} = \frac{(1 - r_2^2)r_3^3}{\tau_L r_2^2} \beta \] (2.10)

### 2.2.2 Polarization Rotated Optical Feedback

Based on the Lang-Kobayashi equations we can build our model for the PROF with 2 modes. The modes are TE and TM, so both will have a equation with a delayed feedback term. The new equations are eqs.2.11 and 2.12. One consideration regarding the coupling strength is that it is considered the same for both modes. We also account for the phase difference \( \phi_i \) between modes through the delay time. Since now we have two modes, the carriers density equation also changes, to account for this second mode to Eq. (2.13).

\[ \frac{dE_{\text{TE}}(t)}{dt} = \frac{1}{2} (1 + i\alpha) \left( G_{\text{TE}}(n(t) - n_0) - \gamma_{ph,\text{TE}} \right) E_{\text{TE}}(t) + k_{\text{inj}} E_{\text{TM}}^+(t - \tau) e^{-i(\omega \tau + \phi_{\text{TE}}(t) - \phi_{\text{TM}}(t - \tau))} \] (2.11)
\[
\frac{dE_{TM}(t)}{dt} = \frac{1}{2}(1 + i\alpha) \left( G_{TM}(n(t) - n_0) - \gamma_{ph,TM} \right) E_{TM}(t) \\
+ k_{inj} E_{TE}(t - \tau) e^{-i(\omega \tau + \phi_{TM}(t) - \phi_{TE}(t - \tau))} \tag{2.12}
\]

\[
\frac{dn(t)}{dt} = J - \gamma_s n(t) - (n(t) - n_0) \left( G_{TE} |E_{TE}|^2 + G_{TM} |E_{TM}|^2 \right) \tag{2.13}
\]

### 2.3 Frequency detuning

The inclusion of the detuning in the model is done by introducing in the equation for the TM mode the term \(-i2\pi\Omega E_{TM}(t)\). \(\Omega\) is frequency detuning. The TE equation and the carriers density equations remain the same. Equations become:

\[
\frac{dE_{TE}(t)}{dt} = \frac{1}{2}(1 + i\alpha) \left( G_{TE}(n(t) - n_0) - \gamma_{ph,TE} \right) E_{TE}(t) \\
+ k_{inj} E_{TM}(t - \tau) e^{-i(\omega \tau + \phi_{TE}(t) - \phi_{TM}(t - \tau))} \tag{2.14}
\]

\[
\frac{dE_{TM}(t)}{dt} = \frac{1}{2}(1 + i\alpha) \left( G_{TM}(n(t) - n_0) - \gamma_{ph,TM} \right) E_{TM}(t) \\
+ k_{inj} E_{TE}(t - \tau) e^{-i(\omega \tau + \phi_{TM}(t) - \phi_{TE}(t - \tau))} - i2\pi\Omega E_{TM}(t) \tag{2.15}
\]

\[
\frac{dn(t)}{dt} = J - \gamma_s n(t) - (n(t) - n_0) \left( G_{TE} |E_{TE}|^2 + G_{TM} |E_{TM}|^2 \right) \tag{2.16}
\]

Alongside the rate equations we show the constants used for the simulations in the following table (Table 2.1) [26]. The parameters varied along the simulations are: the differential gain of the TM mode \((G_{TM})\), the injection coupling strength \((k_{inj})\), and the frequency detuning \((\Omega)\). The modification of the differential gain of the TM mode responds to the interest in the study of the influence of the frequency detuning on the model depending if we use a EEL or a VCSEL. EEL have different differential gainS for the TE and TM modes, and will be characterized by the simulations where \(G_{TE} > G_{TM}\) (by a ratio approx. 6 : 5). The varying of the coupling strength responds to different real situations where one of the parameters related to \(k_{inj}\) in Eq. (2.10) changes (being very easy to change one of the reflectivities). The values of the frequency detuning used rely on the study of its influence in extreme conditions so we can appreciate the changes on the dynamics of the system. The frequency detuning appeared in a system may arise from construction specifics of from external interactions with the device. The value of \(40 \text{GHz}\) can be considered as a proper high value of frequency detuning to study the influence of it in a semiconductor laser under PROF conditions.
Table 2.1: Parameters values used in for the numerical simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{TE}$</td>
<td>Differential gain for the TE mode</td>
<td>$1.374 \times 10^{-12} \text{s}^{-1} \text{m}^3$</td>
</tr>
<tr>
<td>$G_{TM}$</td>
<td>Differential gain for the TM mode</td>
<td>$1.154 \times 10^{-12} \text{s}^{-1} \text{m}^3$</td>
</tr>
<tr>
<td>$\gamma_{ph,TE}$</td>
<td>TE mode photons decay rate</td>
<td>$8.913 \times 10^{11} \text{s}^{-1}$</td>
</tr>
<tr>
<td>$\gamma_{ph,TM}$</td>
<td>TM mode photons decay rate</td>
<td>$8.913 \times 10^{11} \text{s}^{-1}$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency of the TE mode</td>
<td>$1.226 \times 10^{15} \text{s}^{-1}$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>External delay time</td>
<td>$6.670 \times 10^{-9} \text{s}$</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Carriers decay rate</td>
<td>$4.902 \times 10^8 \text{s}^{-1}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Linewidth enhancement factor</td>
<td>3.0</td>
</tr>
<tr>
<td>$n_0$</td>
<td>Carriers density at transparency</td>
<td>$1.400 \times 10^{24} \text{m}^{-3}$</td>
</tr>
<tr>
<td>$J_{th}$</td>
<td>Threshold current density</td>
<td>$1.004 \times 10^{33} \text{m}^{-3} \text{s}^{-1}$</td>
</tr>
</tbody>
</table>

2.3.1 Integration algorithm

The results presented are originated from numerical integration of the equations in Sec. 2.3. Integration is done separating the components of the electric field. The field is separated in its real and imaginary parts ($E(t) = E_R(t) + iE_I(t)$) instead of separating the field in amplitude and phase ($E(t) = \hat{E}(t)e^{i\phi(t)}$). Doing so we avoid problems in the numerical simulation due to low amplitude values. The integration could also be done using complex variable. The algorithm used for the integration is the Runge-Kutta method improved with Gill’s method [37, 38]. The original Runge-Kutta method (RK) is a computing method for step-by-step integration of differential equations. Using as time integration step $h$, the error of RK is of the order $O(h^5)$ at every step. The Gill’s method improves the algorithm because achieves to save information about how much round-off error is made at certain steps in the computation, and makes use of this information to reduce the overall round-off error [39, 40]. The algorithm is:

$$y_{n+1} = y_n + \frac{h}{6} \left( k_1 + (2 - \sqrt{2}) k_2 + (2 + \sqrt{2}) k_3 + k_4 \right) + O(h^5) \quad (2.17)$$

where

$$k_1 = f(x_n, y_n) \quad (2.18)$$

$$k_2 = f \left( x_n + \frac{1}{2} h, y_n + \frac{1}{2} k_1 \right) \quad (2.19)$$

$$k_3 = f \left( x_n + \frac{1}{2} h, y_n + \frac{1}{2} \left( -1 + \sqrt{2} \right) k_1 + \frac{1}{2} \left( 2 - \sqrt{2} \right) k_2 \right) \quad (2.20)$$
\[ k_4 = f \left( x_n + h, \, y_n + \frac{1}{2} \left( -1 + \sqrt{2} \right) k_1 + \frac{1}{2} \left( 2 - \sqrt{2} \right) k_2 \right) \] (2.21)

The subscript \( n \) represents the value for the \( n \)-iteration. In our simulations the variable \( x \) is time, as \( t_n \). The function \( y \) can represent any of our 5 variables, say \( E_{R,TE}, \, E_{I,TE}, \, E_{R,TM}, \, E_{I,TM} \) & \( n \), once separating real and imaginary part in Eqs. 2.14, 2.15. So this algorithm is applied at every iteration for every one of the equations of the model. The time step used in the integration method is \( 8 \times 10^{-14} \) s because it needs to be smaller than the inverse of photon decay rate.

Another point are the initial conditions of the simulation. In a model with noise, the initial conditions could be 0. The system would be in an unstable state and the noise would drive the system outside it towards the stable state. The initial conditions for the electric fields are chosen different from 0 since the model is noise-less, making the system start from the beginning outside the unstable fixed point. The carrier density initial condition is also different from 0. Since grows the slowest, doing so we help the system to start to emit faster. In systems with delayed behaviour the initial conditions at \( t = 0 \) s are not enough. To evaluate future times it is necessary to know not also the value of the variables at \( t = 0 \) s but in the full interval from \( t = -\tau \) s to \( t = 0 \) s (where \( \tau \) is the delay time). So the integration method requires not just 1 initial value for every variable, but \( \tau/h \) values for each variable. The initial conditions used in this work are 0 for \( t < 0 \) s and different from 0 at \( t = 0 \) s. The values of the real and imaginary components of the electric fields are the same for both modes.
Chapter 3

Results of the numerical simulations

3.1 Introduction

The results obtained in the numerical simulations are shown through temporal traces, optical spectra, and cross-correlation plots. The temporal traces show how the optical power of the modes change within a temporal window, small enough to appreciate the dynamics. In the optical spectra we show the spectra for both modes. The optical spectra shown is the results of doing a long simulation and removing the initial part to discard any possible transient regime. Then is made 10 pieces out of the remaining data. After that we calculate the optical spectra of those 10 pieces separately and average them to obtain an average optical spectra. The cross-correlation plots present the cross-correlation between modes in a time scale relative to the delay time (1 = 1 delaytime = 6.67 ns, 2 = 2 delay times, etc). The cross-correlation tells us how much are related two signals with a specific time difference, and it is defined as follows.

\[ C_{TE,TM}(\tau) = \frac{\langle P_{TE}(t) - \hat{P}_{TE} \rangle \langle P_{TM}(t - \tau) - \hat{P}_{TM} \rangle}{\sigma_{TE}^2 \sigma_{TM}^2} \]  

(3.1)

3.2 Influence of the detuning

In this section we present simulation results for the model presented in Ch.2. Our goal is to compare the results obtained with and without frequency detuning between modes. The parameters used for the simulations are mentioned alongside the results, where the detuning (Ω) and the coupling strength (k_{inj}) vary.

The parameters used in the model are those mentioned in Table I of Ch.2. Throughout this chapter, the injection current current is \( J = 1.1J_{th} \). The coupling strength is \( k_{inj} = 75 \text{ ns}^{-1} \).
This value lies within the strong injection range. The difference in differential gain between modes (Te and TM modes) makes the emission in the mode with lower gain (TM in this case) more difficult. Nevertheless the high coupling strength helps emitting in the TM mode due to the injection of the TE mode. We expect the detuning to change the dynamics of the TM mode at least in a qualitative way. The strong coupling gives us a frame of reference with enough visibility of the TM mode to be used for comparison in a wide range of situations. In Sec. 3.2.1 the detuning is neglected, 0 Hz. But in Sec. 3.2.2 the detuning applied is 40 GHz. While this value of detuning is high, detunings of several GHz can be found, tipically in VCSELs.

3.2.1 Results for the case of zero detuning

The simulations in this section are for zero detuning, $\Omega = 0$, and coupling strength of $k_{inj} = 75 \text{ ns}^{-1}$. First we present a temporal trace (Fig. 3.1) and the optical spectra of both modes (Fig. 3.2), and finally a cross-correlation calculation between the two modes (Fig. 3.3).

Figure 3.1: Temporal traces of TE and TM modes under PROF and zero detuning.
3.2.2 Non-zero detuning results

The simulations in this subsection are for a detuning $\Omega = 40\,\text{GHz}$ and coupling strength of $k_{\text{inj}} = 75\,\text{ns}^{-1}$. First we present a temporal trace (Fig. 3.4) and the optical spectra of both modes (Fig. 3.5), and finally a cross-correlation calculation between the two modes (Fig. 3.6).
3.2.3 Discussion of the results

In this subsection we discuss the results obtained from the simulations regarding the value of detuning. First we comment on the dynamics present in the temporal traces. In the output power for the TE mode for $\Omega = 0 \, GHz$ short bursts of energy in the shape of narrow peaks can be seen. These peaks have heights up to $35 \, \mu W$, and mean values around $20 \, \mu W$. The peaks are thin and spaced in time by about $1 \, ns$. In the dynamics of the TE mode for the case of $\Omega = 40 \, GHz$ peaks with height up to $14 \, \mu W$ and mean values of about $6 \, \mu W$ can be seen. There are more peaks in the same time window (almost double), and present wider bases (they
Chapter 3. *Simulation results*

The TM mode power dynamics reveals for $\Omega = 0 \text{GHz}$, few peaks with maximum height of about $10 \mu W$, but with multiple peaks below $5 \mu W$. With the $40 \text{GHz}$ detuning, the number of peaks in the output power remains similar to the case of 0 detuning although with smaller power (lower than $0.5 \mu W$). In general we see a more constant activity in the TE mode output as detuning is applied as compared to the more localized activity in time bursts for the case of zero detuning. For the TM mode the high peaks disappear, remaining the low power peaks.

The optical spectra also change due to the detuning. The FWHM (Full Width at Half Maximum) reduces with increasing detuning. For the case of zero detuning we compute a FWHM of about $50 \text{GHz}$ while for a detuning of $40 \text{GHz}$ our results for FWHM is of about $25 \text{GHz}$, i.e. a reduction of about 50%. The computed power spectra also show widths within 3 orders of magnitude difference between top the and bottom of the spectra. We find that the TE spectra is wider than TM spectra, due to the fact that the TE mode oscillates with more power. In general the difference between TM mode and TE mode spectra is smaller when the detuning is $0 \text{GHz}$ than when the detuning is $40 \text{GHz}$. The detuning reduces the TM spectra, and increases the TE spectra. This indicates that power is more concentrated in the TE mode than in the TM mode compared to the situation of zero detuning. The position of the peaks is also different in the two cases. When the detuning is $0 \text{GHz}$ the peaks are positioned at around $-10 \text{GHz}$ (due to the asymmetry created by the linewidth enhancement factor, $\alpha$) and there is a small difference between the maxima of the TE and TM modes of few $\text{GHz}$. When the detuning is $40 \text{GHz}$ the peaks are positioned closer to $0 \text{GHz}$, and there is almost no difference difference between the maxima of the TE and TM modes.

![Figure 3.6: Cross-correlation between TE and TM mode under feedback and detuning.](image)
When evaluating the cross-correlation we also find differences. The maximum value of cross-correlation is higher for 0 GHz detuning than for 40 GHz detuning, so TE-TM correlation decreases in the presence of detuning. Also the second maximum peaks has smaller height when detuning is present. The peaks are positioned at odd multiples of the time delay, with maximum correlation, with and without detuning, at time equal 1 delay time. So they are more correlated due to the re-injection coupling at the delay time. The situation for detuning the 0 GHz reveals peaks of anti-correlation with very low values (but still noticeable among fluctuations) for even multiples of the time delay. These peaks are not observed if the detuning is 40 GHz.

In general we find that the presence of large detuning alters the dynamics of the TM mode. In this case, less power is distributed to the TM mode and the correlation among modes is reduced in general, due to a smaller interaction between modes. We observe that a large detuning reduces or even destroys the interaction between modes. The power of the TM mode without detuning and feedback would be 0. The cross-feedback transfers some power from the TE mode to the TM mode and consequently the TM mode emits some power. The inclusion of a detuning reduces the power emitted since the interaction due to the feedback is reduced because the modes have different frequencies. This is observed in the optical spectra. The lack of interaction decreases the power available for the TM mode (and leaving more power to the TE mode) leading to a vertical spectra separation. The lowering of the spectral is not homogeneous. Spectra components on the sides of the TM mode peak lose more power. This leads to a narrower TM mode peak on the spectrum. The cross-correlation shows this lack of interaction with lower peaks at odd times. This means that the correlation between modes is not as high as in the absence of detuning, so large detuning reduces their correlation.
3.3 Parameter dependence of the TE-TM dynamics

In this section we present results for the model presented in Ch.2, Eq. (2.11-2.17) modifying some parameter values to observe the dynamics in different situations. The parameters used in the model are still those mentioned in the table of Ch.2 except for the differential gain. We explore 8 different parameter sets. These situations are generated changing 3 parameters, where every parameter has 2 possible values.

The parameters we change are the differential gain, the coupling strength and the detuning value. The coupling strength takes the values either $k_{inj} = 30$ or $50 \, \text{ns}^{-1}$. These two values represent 2 different levels of inter-mode coupling and interaction. The detuning takes the values either $\Omega = +40$ or $-40 \, \text{GHz}$. The differential gain for the TE mode remains the same, $G_{TE} = 1.374 \times 10^{-12} \text{s}^{-1}$, but we change the TM mode differential gain. In particular its value can be the previous one ($G_{TM} = 1.154 \times 10^{-12} \text{s}^{-1}$) or the same as the TE mode ($G_{TM} = 1.1345 \times 10^{-12} \text{s}^{-1}$). The situation where gains are different represent the case of an edge emitting diode (where the low gain of a mode ends up vanishing it). The situation where gains are equal represent the situation in VCSELs (where both modes can emit).

Through the comments of the results of this section we will first compare results when the frequency detuning changes its sign keeping the gain and injection level fixed. Subsequently, we discuss other combinations of parameters.
3.3.1 $G_{TE} > G_{TM}$, $k_{inj} = 30 \text{ ns}^{-1}$, $\Omega = +40 \text{ GHz}$

**Figure 3.7:** Temporal traces of TE and TM mode under low feedback and positive detuning.

**Figure 3.8:** Optical spectra of TE and TM mode under low feedback and positive detuning.

**Figure 3.9:** Cross-correlation between TE and TM mode under low feedback and positive detuning.
3.3.2 $G_{TE} > G_{TM}$, $k_{inj} = 30 \text{ ns}^{-1}$, $\Omega = -40 \text{ GHz}$

**Figure 3.10:** Temporal traces of TE and TM mode under low feedback and negative detuning.

**Figure 3.11:** Optical spectra of TE and TM mode under low feedback and negative detuning.

**Figure 3.12:** Cross-correlation between TE and TM mode under low feedback and negative detuning.
3.3.3 Discussion of the results considering different gains and low coupling strength.

In temporal traces Fig. 3.7 and Fig. 3.10 the TE mode oscillates while the TM mode remains almost silent. This is due to the difference of gain, suppressing this way the mode with lower gain. The peaks of the TE mode appear with a certain regularity that is broken when the sign of the detuning is changed, leading to a more inhomogeneous distribution. A negative detuning seems to induce the TM mode to oscillate strongly since the TM mode peaks are more frequent and higher along time. The optical spectra for the TM mode, Fig. 3.8 and in Fig. 3.11, appear centered close to \(0\,GHz\). The optical spectra for the TE mode do not seem to change when the detuning is changed. The detuning mostly affects the TM mode. The difference between spectra TE and TM modes also reduces if the detuning is set to negative values. In the cross-correlation plots, Fig. 3.9 and Fig. 3.12, it can be seen a big increase in the correlation of the temporal traces. When the detuning is positive the cross-correlation is the highest at one delay time, although its absolute value is still low. When the detuning is negative the correlation between modes increases to almost 1 at one delay time while lower values of correlation occur at odd delay time values. This is a signature that the TM mode follows the TE mode after one delay time.
3.3.4 \( G_{TE} > G_{TM} \), \( k_{inj} = 50 \, ns^{-1}, \, \Omega = +40 \, GHz \)

**Figure 3.13:** Temporal traces of TE and TM mode under moderate feedback and positive detuning.

**Figure 3.14:** Optical spectra of TE and TM mode under moderate feedback and positive detuning.

**Figure 3.15:** Cross-correlation between TE and TM mode under moderate feedback and positive detuning.
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3.3.5 \( G_{TE} > G_{TM}, \ k_{inj} = 50 \text{ ns}^{-1}, \ \Omega = -40 \text{ GHz} \)

**Figure 3.16:** Temporal traces of TE and TM mode under moderate feedback and negative detuning.

**Figure 3.17:** Optical spectra of TE and TM mode under moderate feedback and negative detuning.

**Figure 3.18:** Cross-correlation between TE and TM mode under moderate feedback and negative detuning.
3.3.6 Discussion of the results considering different gains and moderate coupling strength.

The results of increasing the coupling strength to $50 \, ns^{-1}$ are shown in Sec. 3.3.4 and Sec. 3.3.5. The temporal traces, Fig. 3.13 and Fig. 3.16, suffer changes similar to those seen for a lower coupling strength. In this case the coupling strength is stronger, and the TM mode is also more active for negative detuning due to the increase of energy transfer. The spectra in Fig. 3.14 still appear centered near $0 \, GHz$. But more important is the change in the shape of the peaks, for the TE and the TM modes, when detuning is negative shown in Fig. 3.17. The usual “gaussian type peak” turns wider (before, peaks were about $25 \, GHz$ width at FWHM, while now is about $50 \, GHz$) and asymmetrical, showing a left side stepper than the right side. So frequency components on one side are stronger than on the other side. In the cross-correlation we observe for positive detuning, Fig. 3.15, values low correlation between the modes at one delay time (TM mode follows TE mode). When the detuning is negative, Fig. 3.18, the correlation is almost one again for one delay time, and develops other peaks at odd delay times with reduced height as time increases. This shows that the correlation and interaction between the two modes rise for negative detuning.
3.3.7  \( G_{TE} = G_{TM} \), \( k_{inj} = 30 \text{ ns}^{-1} \), \( \Omega = +40 \text{ GHz} \)

**Figure 3.19:** Temporal traces of TE and TM mode under soft feedback and positive detuning.

**Figure 3.20:** Optical spectra of TE and TM mode under soft feedback and positive detuning.

**Figure 3.21:** Cross-correlation between TE and TM mode under soft feedback and positive detuning.
### Chapter 3. Simulation results

#### 3.3.8 \( G_{TE} = G_{TM} , \ k_{inj} = 30 \text{ ns}^{-1} , \ \Omega = -40 \text{ GHz} \)

![Temporal traces of TE and TM mode under soft feedback and negative detuning.](image1)

**Figure 3.22:** Temporal traces of TE and TM mode under soft feedback and negative detuning.

![Optical spectra of TE and TM mode under soft feedback and negative detuning.](image2)

**Figure 3.23:** Optical spectra of TE and TM mode under soft feedback and negative detuning.

![Cross-correlation between TE and TM mode under soft feedback and negative detuning.](image3)

**Figure 3.24:** Cross-correlation between TE and TM mode under soft feedback and negative detuning.
3.3.9 Discussion of the results considering equal gains and low coupling strength.

The results for equal TE and TM gains are interesting for the study of VCSELs. First we discuss the results for low coupling strength (30 $ns^{-1}$) shown in Sec. 3.3.7 and Sec. 3.3.8. In the temporal traces, Fig. 3.19 and Fig. 3.22, it can be seen a situation very similar to those ones commented in the previous paragraph, but now the TM mode is the one prevailing. This is surprising since now the gains are the same, while before the TE mode had bigger gain. Still the peaks are similar. On the optical spectra in Fig. 3.20 we observe now that the TM mode spectrum has a larger amplitude, presenting a shape very similar to the previous TE mode spectrum. Both mode peaks are centered at $-40 GHz$, due to the detuning, showing the prevalence of the TM mode over TE mode again. The peak shift is due to the emitting mode frequency (in this case, TM mode frequency is shifted by $\pm 40 GHz$), while the other one is forced to emit at a frequency close to the dominant one. In the TE mode spectra a new shape can be seen. They are composed by a main peak, an extra single peak for higher frequencies, and a couple of peaks at frequencies close to 0 GHz. The peaks close to the origin remain from the original frequency nature of the TE mode. The high frequency peak, placed around $-60 GHz$, is originated by the oscillations in the electric field noticeable in the temporal traces (for the TE mode). These oscillations are shown in Fig. 3.25, which is a zoom of the trace shown in Sec. 3.3.8. In Fig. 3.25 we can observe peaks with different amplitude, peaks without oscillations, peaks with small oscillations, and a peak with high oscillations. This does not happen when the gain of the TE mode is different from the gain of the TM mode, but occurs when the gains are equal and the detuning different from zero. The frequency of the oscillation is around 60 GHz, originating the peak in the spectrum. The effect of changing the sign of the detuning on the optical spectra (shown in Fig. 3.23) is to flip the spectra around 0 GHz frequency, turning the main peaks at 40 GHz and the fast oscillations peak at 60 GHz. The cross-correlation in Fig. 3.21 and Fig. 3.24 shows similar values. The highest peak is at 0 delay times with a low value of close to 0.25. The second highest peak (height close to 0.1) is present at $-1$ delay times, indicating that now the TE mode follows the TM mode. This change in the correlation as compared with previous results is weakened for negative detuning, but the peak at 0 delay time is slightly increased.
Figure 3.25: TE mode fast oscillations of power with diverse amplitudes.
3.3.10 \( G_{TE} = G_{TM}, \ k_{inj} = 50 \text{ ns}^{-1}, \ \Omega = +40 \text{ GHz} \)

**Figure 3.26:** Temporal traces of TE and TM mode under moderate feedback and positive detuning.

**Figure 3.27:** Optical spectra of TE and TM mode under moderate feedback and positive detuning.

**Figure 3.28:** Cross-correlation between TE and TM mode under moderate feedback and positive detuning.
\subsection*{3.3.11 $G_{TE} = G_{TM}, \ k_{inj} = 50 \ ns^{-1}, \ \Omega = -40 \ GHz$}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.29.png}
\caption{Temporal traces of TE and TM mode under moderate feedback and negative detuning.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.30.png}
\caption{Optical spectra of TE and TM mode under moderate feedback and negative detuning.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.31.png}
\caption{Cross-correlation between TE and TM mode under moderate feedback and negative detuning.}
\end{figure}
3.3.12 Discussion of the results considering equal gains and moderate coupling strength.

The final results from Sec. 3.3.10 and Sec. 3.3.11 are for an increased coupling strength value with respect to the previous results in Sec. 3.3.7 and 3.3.8. In the temporal traces, Fig. 3.26 and Fig. 3.29, for the TM mode peaks with lower minima, with respect to the previous situation, can be seen. Meanwhile the TE mode takes higher values since the coupling strength is higher and there is more power transferred from one mode to the other. In this case it is easier to spot the fast oscillations showed in previous paragraph, noticeable at most TE peaks. They are visible in the temporal trace, Fig. 3.29, as peaks that look like wider lines in comparison with other regions (result of small high frequency oscillations). The optical spectra show the same features shown in Sec. 3.3.7 and Sec. 3.3.8. The TM mode spectrum in Fig. 3.27 is very similar, presenting a small bump for frequencies close to $0 \, GHz$ which was also present before. The spectrum of the TE mode is higher than before due to the increase in the coupling strength in both positive and negative detuning situations. The high oscillations appear stronger for negative detuning (where temporal traces are stronger for TE mode), as peaks close to $0 \, GHz$. On the other hand, in Fig. 3.30 the TE mode peak under the TM main peak (originated by the TM mode coupling) has a smaller height for negative detuning. The cross-correlation plots in Fig. 3.28 and Fig. 3.31 show a small peak for 0 delay times, but a higher (small) peak for −1 delay times, which indicates the TM mode acting over the TE mode.
3.3.13 Discussion of the results

In general we observe a reinforcement of features present when the coupling strength is increased from low to medium feedback. The change of sign in the detuning has significant effects, giving rise to an increase of the cross-correlation when the gains are different. The change of gain has surprising effects. When the gains are different (Sec. 3.3.1 - 3.3.5), the higher gain mode (TE mode here) prevails and controls the smaller gain mode, with centered spectra at the high gain mode frequency (0 GHz). When the two modes have the same gain the detuned mode seems to prevail, switching the emitted frequencies to the detuned one and offering the high frequency oscillations in the suppressed mode. Such situation also shows a switch on the cross-correlation (due to the change of roles) but with lower values. In summary, we observe that the change of the parameters leads to non-trivial dynamical changes in most situations.

After observing the differences between results in different conditions we observe a global effect of the detuning on the level of interaction between the modes depending on the parameters changed. Comparing Sec. 3.3.1 and Sec. 3.3.4 we can see the effect of increasing the coupling strength. The reduced interaction due to the positive detuning is counteracted increasing the coupling strength, leading to an increase of correlation and decreasing the difference in the optical spectra of the modes. The negative detuning causes an interaction between modes more intense than for positive detuning since correlation is higher, but still less than for absence of detuning. Also the optical spectra for the modes are closer and the peaks are wider. More spectral components are active. The situations with equal detuning show an increased interaction also for bigger coupling strengths, even if mostly is destroyed by the detuning, observable through the lack of cross-correlation peaks at odd delay times in $k_{inj} = 30 \text{ ns}^{-1}$. The increase of the coupling strength shows an increase of interaction but small. This interaction is again greater with the negative detuning. So we see how the coupling transfers energy from one mode to the other, but the detuning destroys the existing correlation among them. The negative detuning is less efficient than the positive detuning on removing cross-correlation. In this way the information from one mode is hidden from the other one without significant loss of energy transference.
3.4 Correlation time

In this section we study the evolution of the correlation time as the coupling strength is increased for the last case studied in Sec. 3.3. The correlation time is obtained through the autocorrelation function. The correlation time is the integral of the autocorrelation function for positive delay times. So the correlation time indicates a degree of correlation of the signal with itself. We study the correlation time for one of the cases presented in the previous section. The reason for choosing such case is because its dynamics seems to be the richest. The relevant parameter values are equal gains ($G_{TE} = G_{TM}$) and a negative detuning ($\Omega = -40 \text{ GHz}$). The coupling strength is changed from $k_{inj} = [10\ldots90] \text{ ns}^{-1}$ in intervals of $5 \text{ ns}^{-1}$. Values below $k_{inj} = 10 \text{ ns}^{-1}$ refer to very weak coupling, and therefore low interaction between modes. Values for the coupling strength around $k_{inj} = 90 \text{ ns}^{-1}$ already express high coupling interactions, becoming an adequate maximum for the range. The correlation times for both modes in such conditions are presented in the next figure.

![Figure 3.32: Correlation time evolution for TE and TM modes as coupling strength $k_{inj}$ increases. Note the different scales in the left and right vertical axes.](image)

We can define 3 regions in Fig. 3.32: low coupling strength (from $10 \text{ ns}^{-1}$ to $30 \text{ ns}^{-1}$), medium coupling strength (from $30 \text{ ns}^{-1}$ to $60 \text{ ns}^{-1}$), and strong coupling strength (from $60 \text{ ns}^{-1}$ to $90 \text{ ns}^{-1}$). For each of these regions we plot the autocorrelation and cross correlation for the TE and the TM modes in the following subsections.
### 3.4.1 Low coupling: $k_{inj} = 10 \text{ ns}^{-1}$

**Figure 3.33:** Autocorrelation for the TE mode under low coupling strength.

**Figure 3.34:** Autocorrelation for the TM mode under low coupling strength.

**Figure 3.35:** Cross-correlation for the TE and TM modes under low coupling strength.
3.4.2 Medium coupling: $k_{inj} = 40 \text{ ns}^{-1}$

**Figure 3.36:** Autocorrelation for the TE mode under medium coupling strength.

**Figure 3.37:** Autocorrelation for the TM mode under medium coupling strength.

**Figure 3.38:** Cross-correlation for the TE and TM modes under medium coupling strength.
3.4.3 Strong coupling: $k_{inj} = 80 \text{ ns}^{-1}$

**Figure 3.39:** Autocorrelation for the TE mode under strong coupling strength.

**Figure 3.40:** Autocorrelation for the TM mode under strong coupling strength.

**Figure 3.41:** Cross-correlation for the TE and TM modes under strong coupling strength.
3.4.4 Discussion of the results

In the calculation of the correlation time we have observed clearly 3 regions (see Fig. 3.32), related to low, medium and high coupling strength values. The transition from low to high coupling values reveals a growth of the autocorrelation for the TM mode at even delay times. This is due to the fact that the influence of the TM mode power is transferred to the TE mode after one delay time and back to the TM mode after another delay time. The TE autocorrelation plot presents also an increase in the correlation as the coupling strength increases, but shows peaks at 1 delay time for low and medium coupling strengths, instead of those expected at 2 delay times that start to appear for intermediate levels of coupling. Finally for high values of the coupling strength the autocorrelation has peaks at even delay times.

The extra peaks we observe in Fig. 3.34 are residues of the relaxation oscillations of the TM mode since the coupling is not powerful enough to make it fully unstable. The cross-correlation in Fig. 3.35 has a maximum peak at low coupling for 0 delay time followed by smaller peaks at positive times. As shown in Fig. 3.38 for intermediate coupling strengths the peak at 0 delay time reduces while a peak at −1 delay time rises together with signs of future peaks at odd delay times (sign of how the TM mode is superior on controlling TE mode dynamics over their original dynamics). For high coupling strength we see a series of small peaks at odd delay times with anti-correlation peaks (peaks with negative correlation value) at even times because of the TM mode control over TE mode.

We observe an increase of autocorrelation, as coupling increases, at even delay times since the effect of one mode comes back to it after 2 delay times. In addition the cross-correlation between the TE and TM modes increases as coupling strength increases because the interaction between modes becomes more relevant. Overall, the cross-correlation values appear small due to the detuning present in the system. So we observe right from the beginning a decreased interaction between modes due to the detuning. This interaction increases (autocorrelation reaching high values) while cross-correlation remains as low peaks at odd times but creating correlation at further times. Autocorrelations are affected twice by the detuning effect on the dynamics, presenting this way low correlation values until coupling is strong.
Chapter 4

Final conclusions

In this work we have characterized the response of a two modes semiconductor laser under polarization rotated optical feedback with frequency detuning for different conditions.

In Sec. 3.2 we have simulated the system with and without frequency detuning. The simulation results were shown as temporal traces and optical spectra for the TE and TM modes, and calculation of cross-correlation between the modes. We have observed in the results the decrease of interaction between modes when the detuning is applied. The transference of energy from the TE mode to the TM mode is then reduced, showing reduced peaks in TM mode temporal trace and lower TM mode spectra. The cross-correlation is affected with lower values overall. These results correspond to the typical difference of modal gain found in EELs.

In Sec. 3.3 we have simulated the system with frequency detuning modifying the parameter values to observe the dynamics at different situations. The parameters modified are the sign of the detuning, the coupling strength, and the gain of the TM mode. In general we observe that the interaction between TE and TM modes results from competition between the coupling strength defined for the amount of feedback reinjected and the detuning. The detuning modifies the optical spectra of the TE and TM modes stretching them. We observe for equal gains a dominance of the mode with detuning in the temporal traces as output power independently of the sign of the detuning, and a major influence of the lack of interaction between them about information through a strong reduction of cross-correlation. The influence of the detuning appears less significant when the sign of the detuning is negative. The results for equal gains corresponds to typical parameters found in VCSELs.

In Sec. 3.4 we have simulated the system with detuning modifying the coupling strength parameter over a range for certain values of detuning and gains. We have calculated the correlation time in that set of values. Recognising three regions, we have calculated the autocorrelation function for the TE and TM modes and the cross-correlation function between them in a set of
values belonging to those regions. This allows us to see the correlation behaviour qualitatively in every region. In the cross-correlations we observe the reduced interaction between modes at odd multiples of the delay time as coupling strength increases, showing a reduced effect of the information carried by a mode onto the other. The autocorrelations show an increase of correlation for even multiples of the delay time, so the modes can still affect themselves even though the information has passed through the other mode.

In this work we have investigated the detuning in extreme conditions. Remains unknown the behaviour of the system in intermediate conditions under influence of frequency detuning. Frequency detuning influence on the dynamics of the system can vary along the range of values leading to unexpected results. This work can be used to support future works on the study of frequency detuning influence in laser under polarization rotated optical feedback with other parameters such other mode gains ratio, high injection current, or even higher coupling injection strength between modes. Each one of these situations, as the ones studied in this work, relate to possible real scenarios. The study of situations where mode gain is equal is related to the use of VCSELs as a light source, where the study of the model with different mode gain relates to the use of EELs. For applications that require high intensity, conclusions from high currents studies is necessary. And conditions where coupling injection can be obtained experimentally just changing the reflectivity of the mirror of the external cavity, recreating the conditions of semiconductor laser receiving feedback from a surface with higher reflectivity. For now we extracted theoretical results from simulations. Experimental work of this model is one of the possible next steps. Experimental investigation of these characteristic results and the high frequency oscillations observed in Sec. 3.3 can be the next project. We see the present work arises many questions with no easy answers within a wide range of possibilities. Answering all of them would require much more time and effort than the one we have on the present day. But for sure some of the answers will be interesting and surprising, showing to us that the field of semiconductor lasers under optical feedback is far richer than what anyone would have ever imagined.
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