Meeting on Relativistic Quantum Walks

February 6,7 2014, Université de Grenoble

Local particles in QFT

Local quanta, unitary inequivalence, and vacuum entanglement

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QUINFOG

Quantum Information and Foundations Group

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YouTube channel

Quinfog has now a YouTube channel

Local Quantum Theory

Quantum Information and Quantum Fields at Facultad de Físicas. 2nd, 3rd and 4th December!!

SUMMARY

IF IT IS NOT THERE, IS IT ZERO?

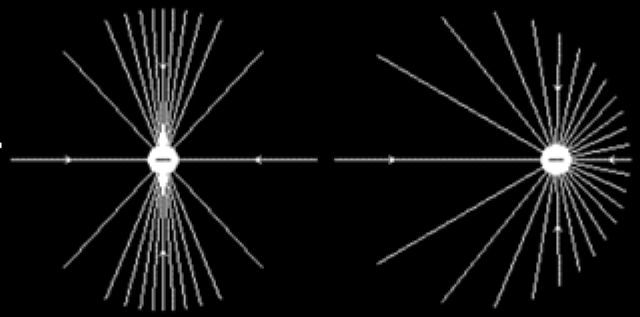
NO!

IF IT IS NOT THERE, THERE IS THE VACUUM

but……

This leaves no way for strict localization

Madrid 1967 *Juan:* What is an electron?
EM LabTeaching Assistant: Something like this ………….



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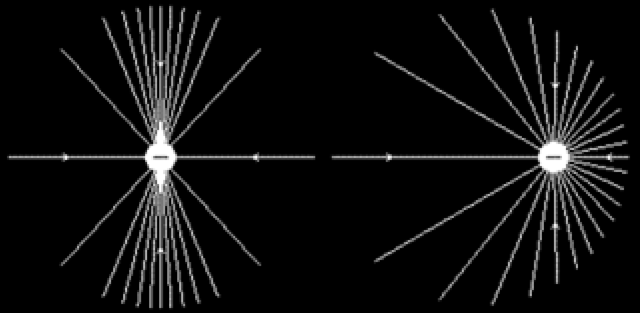

Charged particle as a point creating a Coulomb field

Classical Charged Particles (Lorentz, Abraham, Rohrlich, Dirac,..)

Particles follow worldlines

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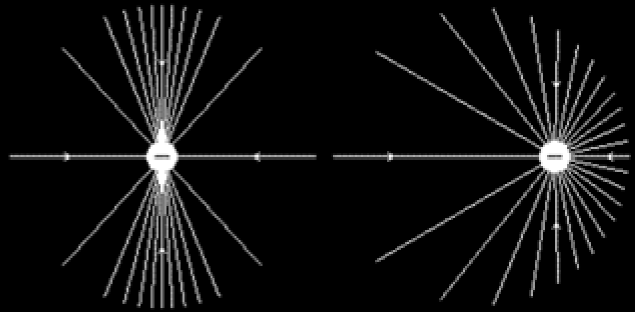

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The transplant of this idea to QM is best represented by Bohmian Mechanics

Beyond that

- $q \to \hat{q}, \;\; p \to \hat{p}, \{q,p\} \to [\hat{q},\hat{p}]$
- Hamilton-Jacobi → Schrödinger
- $\psi_{\hat{q}=x}(x') = \delta(x'-x), \;\; \psi_{\hat{p}=p}(p') = \delta(p'-p)$

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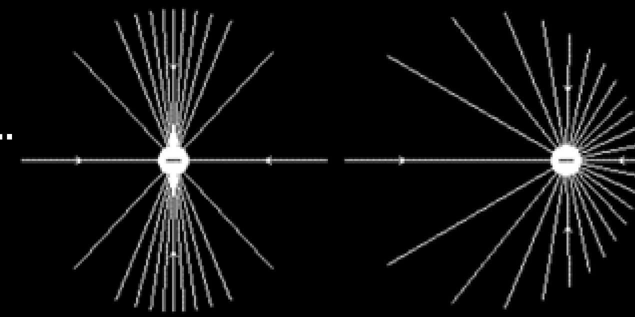

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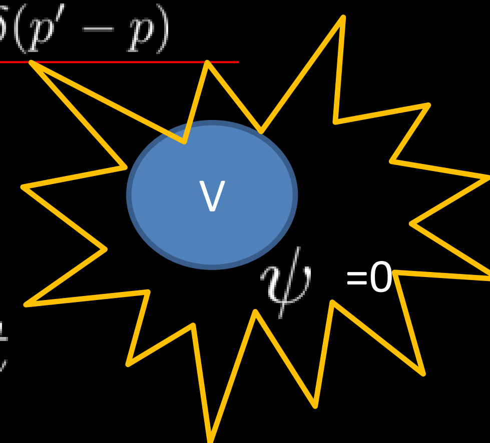
But Hegerfeldt: If at t=0 ψ localized in V and $\sigma(H) > 0$

then ψ is

- forever in V (vanishes out of V)
- finite everywhere after infinitesimal δt



Hegerfeldt Theorem

$$\psi_t = e^{-iHt}\psi_0 \qquad \hat{H} \geq c$$

$$\mathcal{P}_A(t) = \langle \psi_t | A | \psi_t \rangle \quad \hat{A} \geq 0$$

$$\Rightarrow \begin{cases} Either\ \mathcal{P}_A(t) \neq 0 \, \forall\, t \in \mathbf{R} \\ or\ \mathcal{P}_A(t) = 0 \, \forall\, t \in \mathbf{R}. \end{cases}$$

R. Paley and N. Wiener Theorem XII

Take $\boxed{A = \int_V |x\rangle\langle x|}$ V Borel set in $\mathbf{R}^3$, |x > position eigenstate

$$Either\ \psi \text{ is in } V \text{ forever } (P_V(t) \neq 0 \, \forall\, t \in \mathbf{R})$$

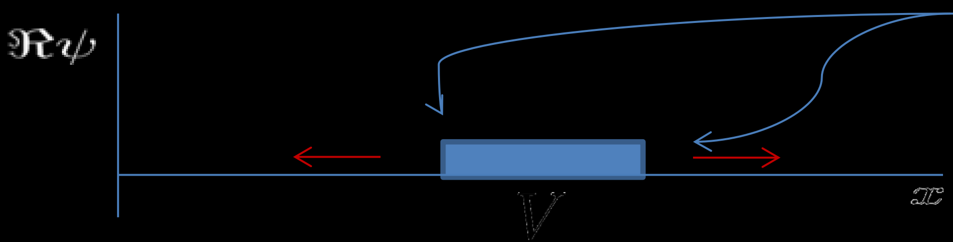
$$or\ \psi \text{ is never in } V \ (P_V(t) = 0 \, \forall\, t \in \mathbf{R})$$

Instantaneous spreading $\Longrightarrow$ causality problems in RQM and QFT

Prigogine: KG particle $|\psi\rangle = \int_0^\infty d\omega_k \psi(\omega_k, x) a^\dagger(k)|0\rangle,$

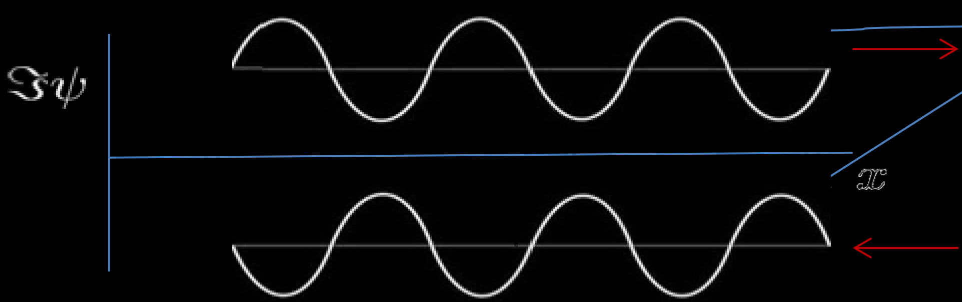
initially localized in $V, \quad \psi(\omega_k, x) = 0 \text{ if } x \notin V$

box splits and expand at the speed of light



$$\boxed{\begin{gathered}\Box\psi(t,x) = 0 \\ \psi(t,x) = \Re\psi(t,x) + i\Im\psi(t,x)\end{gathered}}$$

destructive interference at $t = 0$



$\Im\psi(t,x)$ strictly non local

Antilocality of $\hat{\omega}_k : \psi(t,x) = 0, \text{ and } \hat{\omega}_k\psi(t,x) = 0 \forall x \in I \Rightarrow \psi(t,x) \equiv 0$

A simplified version of Ree-Schlieder th.

Even if initially $\psi(0,x) = 0 \ \forall x \notin V$, Necessarily $\dot{\psi}(0,x) \neq 0$ for $x \notin V$

We localized a Fock state
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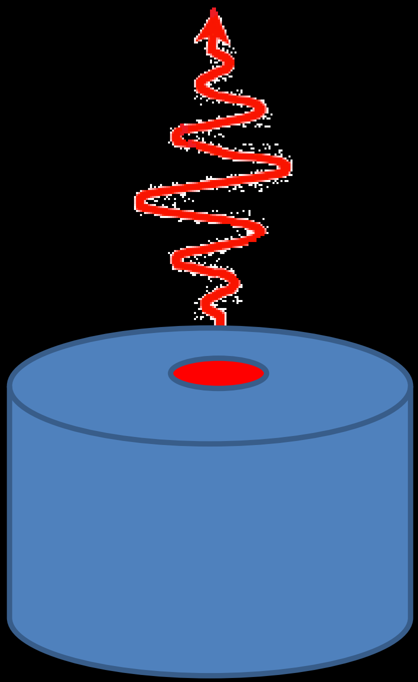
Wave function and its time derivative
vanishing outside a finite region
requires of both, positive and negative ,frequencies

What happens when a photon,

produced by an atom inside a cavity,

escapes through a pinhole?



Eventually the photon will impact on a screen at $t = d/c$

But only at the pinhole $\phi \neq 0$
and the photon energy is positive

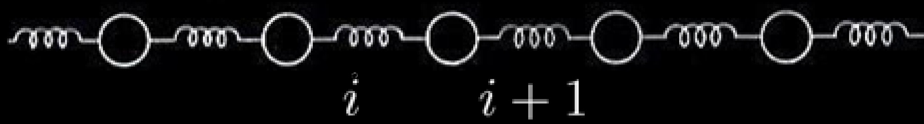
According to Hegerfeld + antilocality
the photon will spread everywhere almost
instantaneously

As this is not the case,

we have to abandon Fock space for describing the photon through the pinhole

Quantum springboards

$$H=\sum_i \tfrac{1}{2}(p_i^2+q_i^2)+\lambda q_i q_{i+1} \quad \rightarrow \quad H=\sum_i \tfrac{1}{2}(P_i^2+Q_i^2)$$



(q_i, p_i)local d.o.f. $\qquad\qquad$ (Q_i, P_i)global d.o.f.

$$\hat{A}_N=\hat{Q}_N+i\hat{P}_N,\ \ \hat{A}_N^\dagger=\hat{Q}_N-i\hat{P}_N, \rightarrow H=\sum_N \hbar\Omega_N \hat{A}_N^\dagger \hat{A}_N$$

Particles $\longleftrightarrow$ elementary excitations of global oscillators

Vacuum $\longleftrightarrow$ ground state

All oscilators are present in the vacuum

Local excitations are not particles, Global are (standard Fock Space)

Vacuum entanglement: what you spot at q_i depends on q_j

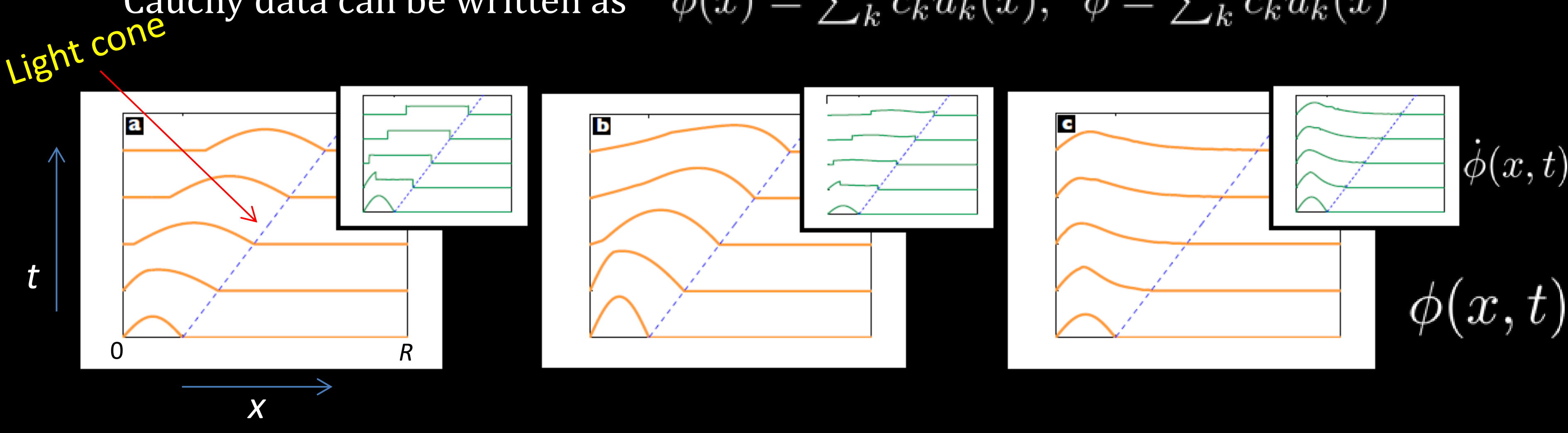
We need to describe the photon emerging through the pinhole as a well posed Cauchy problem i.e. by initial values fo ϕ and $\dot{\phi}$ vanishing outside the hole.

Consider a 1+1 Dirichlet problem where the global space i $\{x \in [0, R]\}$

and the initial data vanish outside $L = [0, r],\ r < R$

We will consider $u_k(x) = \frac{1}{\sqrt{r\omega_k}} \sin \frac{\pi k x}{r}, \quad \dot{u}_k(x) = -i\omega_k \frac{1}{\sqrt{r\omega_k}} \sin \frac{\pi k x}{r}, k = 1, 2..$

Cauchy data can be written as $\phi(x) = \sum_k c_k u_k(x), \quad \dot{\phi} = \sum_k \dot{c}_k \dot{u}_k(x)$



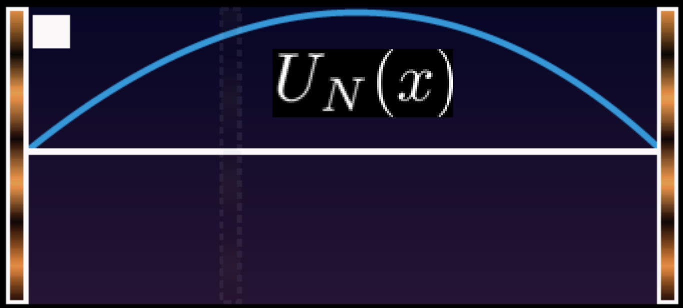
We follow a similar procedure for the case finite Cauchy data out of the hole:

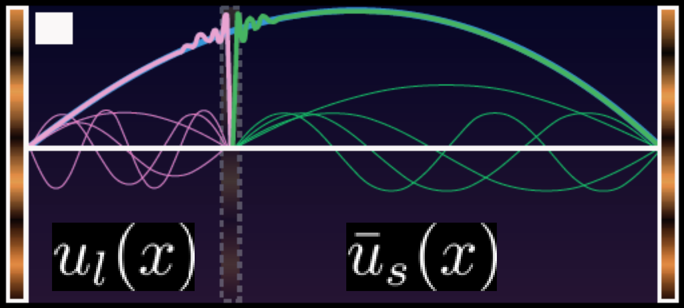
$$x \in \bar{L} = [r, R], \quad \bar{r} = R - r$$

$$\bar{u}_k(x) = \frac{1}{\sqrt{r\bar{\omega}_k}} \sin \frac{\pi k x}{\bar{r}}, \quad \dot{\bar{u}}_k(x) = -i\bar{\omega}_k \frac{1}{\sqrt{r\bar{\omega}_k}} \sin \frac{\pi k x}{\bar{r}}, k = 1, 2...$$

Finally, using upper case for the global modes:

$$U_N(t,x) = U_N(x) e^{-i\Omega_N t} = \frac{1}{\sqrt{R\Omega_N}} \sin \frac{\pi N x}{R} e^{-i\Omega_N t}$$ (*N.B. stationary modes*)





How sums of $u_l(x)$ and $\bar{u}_s(x)$ build up $U_N(x)$

Field expansions

$$\hat{\phi}(x,t) = \sum_{N=1}^{\infty} \left(U_N(x,t)\hat{A}_N + U_N^*(x,t)\hat{A}_N^\dagger \right), x \in R$$

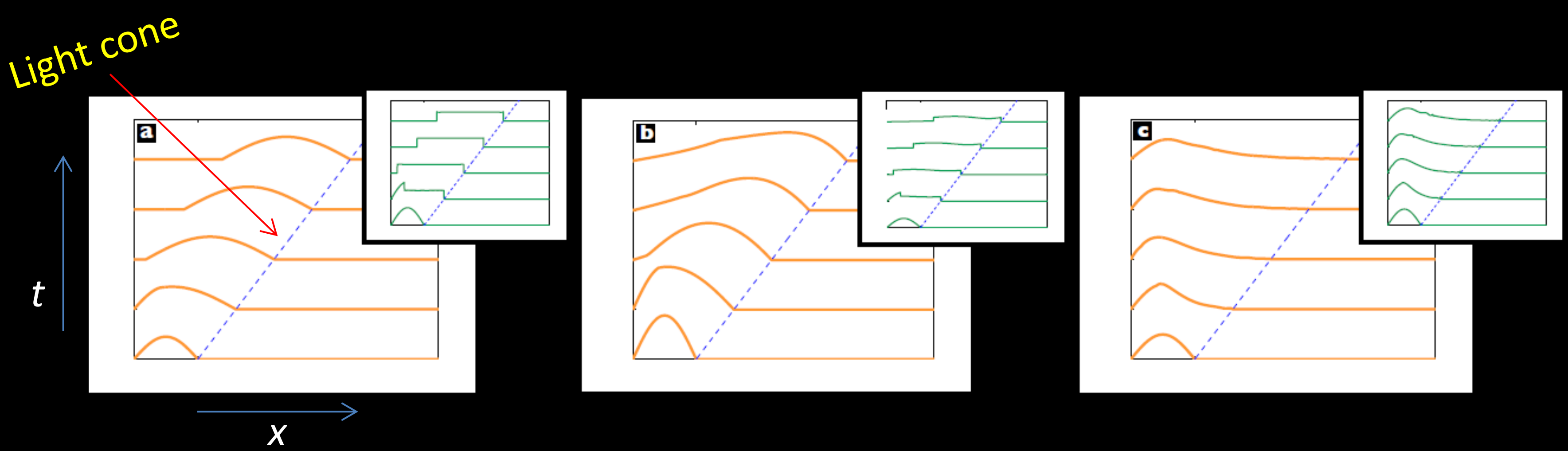
$$\hat{\phi}(x,t) = \sum_l \left(u_l(x,t)\hat{a}_l + u_l^*(x,t)\hat{a}_l^\dagger \right), \quad x \in L$$

$$\hat{\phi}(x,t) = \sum_l \left(\bar{u}_l(x,t)\hat{\bar{a}}_l + u_l^*(x,t)\hat{\bar{a}}_l^\dagger \right), \quad x \in R - L$$

Local modes are superpositions of positive and negative frequencies

$$u_k(x,t) = \sum_N \left((\omega_k + \Omega_N) e^{-i\Omega_N t} - (\omega_k - \Omega_N) e^{i\Omega_N t} \right) \mathcal{U}_N(x) \mathcal{V}_{kN}$$

$$\bar{u}_k(x,t) = \sum_N \left((\bar{\omega}_k + \Omega_N) e^{-i\Omega_N t} - (\bar{\omega}_k - \Omega_N) e^{i\Omega_N t} \right) \mathcal{U}_N(x) \bar{\mathcal{V}}_{kN} .$$



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Bogoliubov transformations

$$a_m = \sum_N (u_m|U_N)A_N + (u_m|U_N^*)A_N^\dagger \qquad a_m^\dagger = \sum_N (U_N|u_m)A_N^\dagger + (U_N^*|u_m)A_N$$

$$\bar{a}_m = \sum_N (\bar{u}_m|U_N)A_N + (\bar{u}_m|U_N^*)A_N^\dagger \qquad \bar{a}_m^\dagger = \sum_N (U_N|\bar{u}_m)A_N^\dagger + (U_N^*|\bar{u}_m)A_N.$$

Canonical conmutation relations

$$[a_m, a_n^\dagger] = \delta_{mn} \qquad [a_m, \bar{a}_n^\dagger] = 0 \qquad [\bar{a}_m, \bar{a}_n^\dagger] = \delta_{mn}$$

Exciting the vacuum with local quanta

$$|0_G\rangle \to a^\dagger_m|0_G\rangle$$

Normalized one-local quantum state

$$|\psi\rangle = \frac{a^\dagger_m|0_G\rangle}{\sqrt{1+\langle 0_G|n_m|0_G\rangle}}$$

If $\left|\psi\right\rangle$ were strictly local,

operators acting on $L-R$

could not tell the difference between $\left|\psi\right\rangle$ and $\left|0_G\right\rangle$

For instance

$$\langle\psi|\bar{n}_m|\psi\rangle - \langle 0_G|\bar{n}_m|0_G\rangle$$

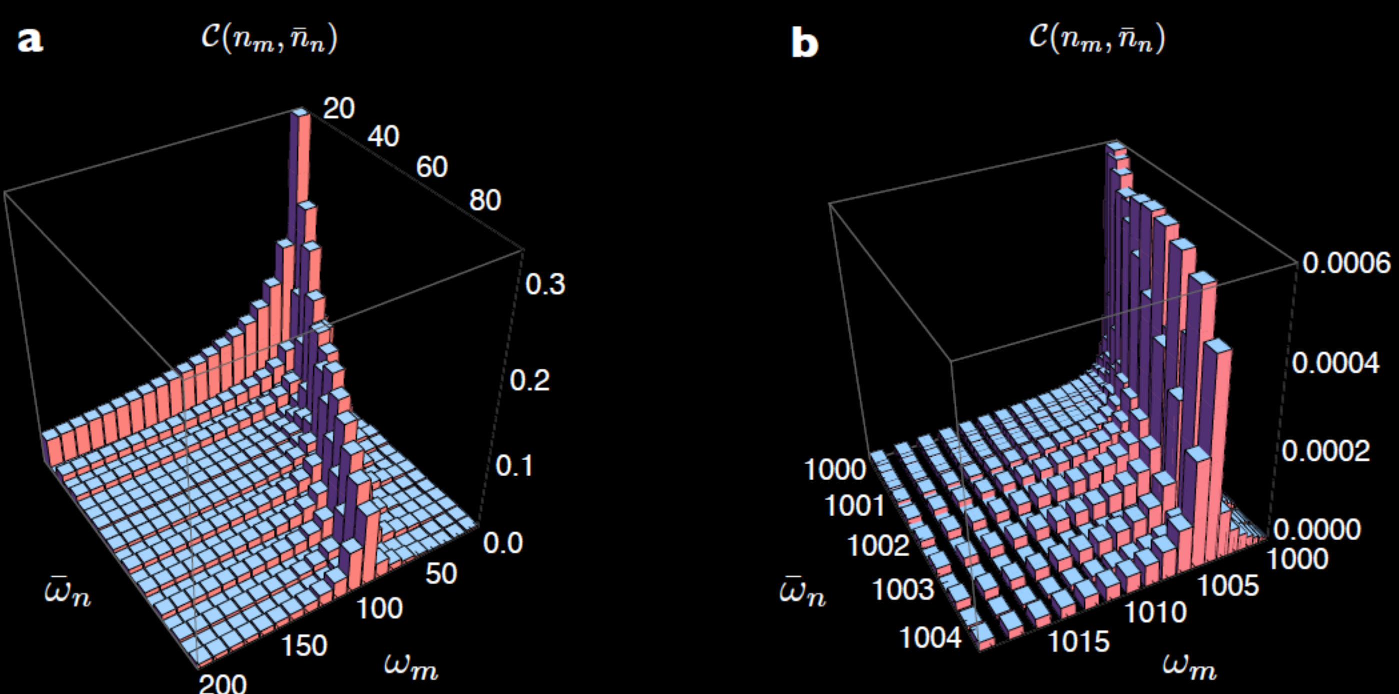
should be zero

This is not the case

$$\langle\psi|\bar{n}_m|\psi\rangle - \langle 0_G|\bar{n}_m|0_G\rangle = \frac{\langle 0_G|a_l\bar{n}_m a_l^\dagger|0_G\rangle}{1+\langle 0_G|n_l|0_G\rangle} - \langle 0_G|\bar{n}_m|0_G\rangle$$
$$= \frac{\langle 0_G|n_l\bar{n}_m|0_G\rangle - \langle 0_G|\bar{n}_m|0_G\rangle\langle 0_G|n_l|0_G\rangle}{1+\langle 0_G|n_l|0_G\rangle} = \frac{corr(n_l,\bar{n}_m)}{1+\langle 0_G|n_l|0_G\rangle}$$

due to vacuum correlations

$$corr(n_m,\bar{n}_l) \equiv \langle 0_G|n_m\bar{n}_l|0_G\rangle - \langle 0_G|n_m|0_G\rangle\langle 0_G|\bar{n}_l|0_G\rangle$$



The lesson:

In spite of having introduced neatly localized quanta

Their elementary excitations are not strictly local

We traced this back to the vacuum non-locality $corr(n_l, \bar{n}_m) \neq 0$

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N.B. We could introduce a local vacuum $\hat{a}_l |0_L\rangle = 0$

then the states $|\phi_L\rangle = \hat{a}^\dagger |0_L\rangle$

Become strictly local $\langle \phi_L | \bar{n}_m | \phi_L \rangle - \langle 0_L | \bar{n}_m | 0_L \rangle = 0$

Thank You!