Mimicking Matter in the Hybrid Quantum Gowdy Models
• LQG as a GR quantum theory \(\rightarrow\) LQC

• Flat FRW model leads to a Big Bounce.

• Simplest inhomogeneous Universe \(\rightarrow\) \(T^3\) Gowdy model with linear polarization.

• Two spatial Killing vectors; solutions with LRS.

• More physical scenario: Minimally coupled massless scalar field. Non zero modes vary in the same direction as gravitational waves.

• Hybrid quantization and approximations.

• Family of states that could mimick exotic matter content in a flat FRW Universe.
Hybrid Quantization of the Gowdy model

- Loop quantization of the homogeneous sector (Bianchi I) minimally coupled to a massless scalar field $\Phi$ is combined with a Fock quantization of gravitational and matter fields varying in one angular direction.

$$\hat{C}_G = \hat{C}_{FRW} - \frac{\pi G \hbar^2}{8} (\hat{\Omega} \hat{\Theta} + \hat{\Theta} \hat{\Omega}) + \frac{2 \pi G \hbar^2}{\beta} e^{2\Lambda} \hat{H}_0 + \frac{\pi G \hbar^2 \beta}{4} e^{-2\Lambda} \hat{D} \hat{\Omega}^2 \hat{D} \hat{H}_I$$

Where

$$\hat{C}_{FRW} = -\frac{3 \pi G \hbar^2}{8} \hat{\Omega}^2 - \frac{\hbar^2 \partial^2 \phi}{2}$$

$$\hat{H}_0 \equiv \sum_{\alpha \in \xi, \varphi} \sum_{m \in \mathbb{Z} - [0]} |m\rangle \langle m| \hat{a}(\alpha) \hat{a}^{\dagger}(\alpha)$$

$$\hat{H}_0 \equiv \sum_{\alpha \in \xi, \varphi} \sum_{m \in \mathbb{Z} - [0]} \frac{1}{2|m|} (2 \hat{a}(\alpha) \hat{a}^{\dagger}(\alpha) + \hat{a}(\alpha) \hat{a}^{\dagger}(\alpha) + \hat{a}(\alpha) \hat{a}^{\dagger}(\alpha) - \hat{a}(\alpha) \hat{a}^{\dagger}(\alpha))$$

$$\beta \equiv [G \hbar / (16 \pi^2 \gamma^2 \Delta)]^{1/3}$$
Approximations: Spectrum of the Flat FRW operator

- Eigenfunctions \( e_{\rho}(v) \) of the geometry, with eigenvalue \( \rho^2 \), are exponentially suppressed for \( v \leq \rho/2 \), specially for \( \rho >> 10 \).

Oscillatory behavior at large values of \( v \).

- \( \hat{D} \) comes from the regularization of the inverse of the volume in LQC.

  It acts by multiplication on \( |v\rangle \) states. Eigenvalue \( D(v) \rightarrow 1 \) when \( v >> 1 \), and \( D(v)=0 \) when \( v=0 \).

- We can restrict ourselves to states with \( \rho >> 10, v >> 1 \).

- We can expand the homogeneous and isotropic part of the states in the \( |e_{\rho}\rangle \) basis.
Approximations: anisotropy and interaction terms

- Consider smooth anisotropy wavefunctions $f(\Lambda)$ that for variations in $\Lambda$ smaller than $q_\varepsilon$ satisfy $f(\Lambda + \Lambda_0) \approx f(\Lambda) + \Lambda_0 \partial_\Lambda f(\Lambda)$

- Then the anisotropy term approximately factorizes:
  $\hat{\Omega} \hat{\Theta} + \hat{\Theta} \hat{\Omega} \approx 2 \hat{\Omega} \hat{\Theta}$, acting on states $|e^{\varepsilon \rho}\rangle \otimes |f(\Lambda)\rangle$, where $\hat{\Omega}'$ is analogous to $\hat{\Omega}$ if $v >> 1$, and $\hat{\Theta}'$ is the discretization of $-4i \partial_\Lambda$ at the scale $q_\varepsilon \geq \log(1 + 4 \rho^*/\rho^*)$, for $\rho \geq \rho^*$.

- We consider Gaussian profiles peaked in $\Lambda$ and its momentum:

$$|\psi_\Lambda\rangle \approx \sum_{\Lambda \in \mathcal{L}} \frac{\sqrt{\sigma_s}}{4\sqrt{\pi}} e^{\frac{\sigma_s^2}{2 q_\varepsilon^2} (\Lambda - \bar{\Lambda})^2} |\Lambda\rangle \text{ with } \sigma_s \ll \pi/2$$

then $\langle \bar{\psi}_\Lambda | \hat{\Theta}' | \psi_\Lambda \rangle = 0$, and $\langle \bar{\psi}_\Lambda | e^{-2\Lambda} | \psi_\Lambda \rangle \ll 1 \text{ if } \bar{\Lambda} >> q_\varepsilon^2 / \sigma_s^2$
Approximate solutions to the Gowdy model

- One arrives at the approximate Hamiltonian constraint:

\[
\hat{C}_{\text{app}} = \frac{-3\pi G \hbar^2}{8} \Omega^2 - \frac{\hbar^2 \partial^2}{2} + \frac{2\pi G \hbar^2}{\beta} e^{2\Lambda} \hat{H}_0
\]

- Solving \( \langle \phi, \nu, \Lambda, n^\xi, n^\Phi | \hat{C}_{\text{app}} | \Psi \rangle = 0 \) for \( \rho \), solutions take the form:

\[
\Psi(\phi, \nu, \Lambda, n^\xi, n^\Phi) = \int_{-\infty}^{\infty} dp_\phi \Psi(p_\phi, \Lambda, n^\xi, n^\Phi) e^{\epsilon(\nu)} e^{\rho_\phi(p_\phi, \phi)}
\]

Where \( \Psi(p_\phi, \Lambda, n^\xi, n^\Phi) = \Psi(p_\phi, n^\xi, n^\Phi) \psi_\Lambda(\Lambda) \), and \( \rho \) satisfies:

\[
\rho^2 = \frac{4}{3\pi G \hbar^2} p_\phi^2 + \frac{16}{3\beta} e^{2\Lambda} H_0(n^\xi, n^\Phi)
\]

- We can focus our attention on states with \( \Lambda = \overline{\Lambda} \), considering that our anisotropy profiles are strongly peaked there.
Peak depending on constants of motion

- Friedmann equation for flat FRW: \( \frac{3}{a^2} \dot{a}^2 = 8 \pi G \omega \)

Comparing with the classical constraint for flat FRW minimally coupled to a massless \( \phi \):

\[
\omega = \frac{p^2}{2a^6} = \left(8 \pi^2 G^2 \hbar^2 \Delta \right)^{-1} \frac{p^2}{\nu^2} \quad \rho^2 = \frac{\dot{a}^2}{a^2} \frac{a^6}{\pi^2 G^2 \hbar^2}
\]

- Setting \( \bar{\Lambda} = \bar{\Lambda}(p^2 \phi, H_0) \) we regard the solution of \( \hat{C}_{app} \) as a corrected flat Friedmann equation, for small content in inhomogeneities:

\[
\rho^2 = \frac{4}{3 \pi G \hbar^2} p^2 + \frac{16}{3 \beta} e^{2 \pi(p^2 \phi, H_0)} \quad H_0(n^c, n^v) \equiv F(p^2 \phi, H_0) \quad \Rightarrow \quad p^2 \phi = F^{-1}(\rho^2, H_0)
\]

If \( H_0 \ll 1 \) and \( \omega_0 = \frac{3 \pi G \hbar^2}{8a^6} \rho^2 = \frac{3}{8 \pi G} \left( \frac{\ddot{a}}{a^2} \right) \):

\[
\omega = \omega_0 + \left(8 \pi^2 G^2 \hbar^2 \Delta \right)^{-1} \frac{1}{\nu^2} \sum_{n=1}^{\infty} \frac{1}{n!} \left[ \frac{\partial^n F^{-1}(\rho^2, H_0)}{\partial H_0^n} \right]_{H_0=0}
\]
Peak depending on the volume

- More physically interesting scenario → Allowing the peak to depend on the volume so it can behave as new matter content.

- Setting \( \Lambda = \Lambda(v, p^2, H_0) \), makes \( \psi_{\Lambda}(\Lambda) \rightarrow \psi_{\Lambda}(\Lambda, v, p^2, H_0) \).

- Are the approximations still valid?

- On the one hand, for anisotropy states \( |f(v, \Lambda)\rangle \) with same properties as the previous ones in their \( \Lambda \) dependence:
  \[ \hat{\Omega} \hat{\Theta} + \hat{\Theta} \hat{\Omega} \approx 2 \hat{\Omega'} \hat{\Theta'} \] still holds, but states no longer separate.

  \[ \langle \psi_{\Lambda} | 2 \hat{\Omega'} \hat{\Theta'} | \psi_{\Lambda} \rangle \neq 0 \text{ now, but } \langle \psi_{\Lambda} | \hat{\Theta'} | \psi_{\Lambda} \rangle = 0 \] holds as well.

Therefore, if \( \Theta' \)'s dispersion is sufficiently small, we may disregard this term compared to \( \hat{\Omega}^2 \).
Peak depending on the volume: Interaction term

- On the other hand, acting on the anisotropy states gives:
  \[e^{-2\Lambda} \hat{\Omega}^2 |\psi_{\Lambda}\rangle = \sum_{v \in \mathcal{L}_e^+} \sum_{\Lambda \in \mathcal{L}} \psi_{\Lambda}(\Lambda, v) e^{-2\Lambda} \hat{\Omega}^2 |\Lambda, v\rangle\]

  Where \(D(v) \rightarrow 1\) holds in the regime \(v \gg 1\).

- \(\hat{\Omega}^2\) basically produces shifts of step 4 in the volume.

- Besides, \(\langle \psi_{\Lambda} | e^{-2\Lambda} | \psi_{\Lambda}\rangle \ll 1\) if \(\Lambda(v) \gg q_e^2/\sigma_s^2\) stands.

We can focus our attention on \(v\) dependences such that \(\Lambda(v)\) increases as the volume grows larger. Then, we may disregard the interaction term compared to \(\hat{\Omega}^2\).

- This kind of dependence in our Friedmann equation would mean that the density corrections would decrease slower than the free scalar field contribution \(\longrightarrow\) Behave as new matter.

- Further analysis of this term may provide more information on the details of the dependence.