QUANTUM CORRELATIONS ACROSS COSMOLOGICAL HORIZONS

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Motivation of this work

- Physical multiverse
  - Independent spacetimes that produce effects which can be observables in our own universe
    - Classical effects
    - Quantum effects
  - Considering models in which the universes could have quantum correlations between each other
    - Quantized system that relate the different single universes

- We study, in the first place, quantum correlations between different regions of the Universe
What do we want to do?

- Our aim is to study the possible influence of other regions of the universe which are not classically accessible.

- We are looking for quantum correlations across horizons.
  - We construct a model of a spacetime divided in different regions.
  - We will find an exact quantization of this spacetime.
  - We will consider physical states that mix the different regions.
  - We want to define an observable that allow us determine quantum contributions in one region coming from the other ones.
Previous works

In the context of canonical quantum cosmology

- **Semiclassical quantization of the extended de Sitter spacetime** (R. Laflamme)
  - Solutions only span one region $\rightarrow$ analytic continuation to the other region
  - It is expected to recover these solutions in the semiclassical limit

- **Quantum wormholes in a Kantowski-Sachs spacetime** (L. M. Campbell and L. J. Garay)

- **Wormholes as a basis for the Hilbert space in Lorentzian gravity** (Guillermo A. Mena Marugán)
➢ Quantization of Schwarzschild black holes (M. Cavaglia et al)

  ▪ Independent quantization of each region

  ▪ No cosmological constant

➢ Quantum mechanics of the Schwarzschild-de Sitter black hole (G. Oliveira-Nieto)

  ▪ Semiclassical approximation

  ▪ Only determine the wavefunction in our region
General spherically symmetric metric that depends on two parameters \((A, b)\) and provided a cosmological constant \((\Lambda)\)

\[
ds^2 = -\frac{N^2(r)}{A(r)} dr^2 + A(r) dt^2 + b^2(r) d\Omega_2^2
\]

where \(0 \leq r < \infty\) and \(-\infty < t < \infty\).

- \(N\) is a Lagrange multiplier
- \((A, b)\) configuration variables \(A \in \mathbb{R}\) and \(b \in \mathbb{R}^+\)
- Cosmological horizons: \(A(r')=0\)
- We have exchange \(r\) and \(t\) respect the usual formulation
- **Einstein-Hilbert action**
  \[ S = \int dr L \]
  where
  \[ L(N, A, b, \dot{A}, \dot{b}) = \frac{1}{2} \left( \frac{b\dot{b}\dot{A}}{N} + \frac{A\dot{b}^2}{N} - N + \Lambda b^2 N \right) \]

- **Classical solutions**
  \[ A = \frac{\Lambda b^2}{3} - 1 + \frac{2m}{|b|} \]
  \[ |\dot{b}| = N \]

- **Penrose diagrams of the classical spacetimes**
m > 0 and $9m^2\Lambda < 1$

**Schwarzschild-De Sitter spacetime**

Other regions are exact copies.

where

$r_b \approx 2m$

$r_c \approx \sqrt{\frac{3}{\Lambda}} - m$

$r = r_b = r_c = \frac{1}{\sqrt{\Lambda}}$
New configuration space

- New configuration variables

\[
\begin{align*}
2m &= Ab - b \left( \frac{\Lambda b^2}{3} - 1 \right) \\
2\tau &= Ab + b \left( \frac{\Lambda b^2}{3} - 1 \right)
\end{align*}
\]

- They allow us to construct a separable Hamiltonian constraint

- But they are multivalued functions of the original variables

- We have to include the **negative values of b** in order to cover the whole space
The action is now given by

\[ S = \frac{1}{2} \int dt N' \left( \frac{\dot{\tau}^2}{N'^2} - \frac{\dot{m}^2}{N'^2} + 1 \right) \]

where \( N' = (\Delta b^2 - 1)N \)

In order to perform a canonical quantization, we are interested in making a Hamiltonian formulation of the system.

- Hamiltonian is given by

\[ H = \frac{N'}{2} (\Pi_\tau^2 - \Pi_m^2 - 1) \]

with:

\[ \Pi_\tau = \frac{\dot{\tau}}{N'} \]

\[ \Pi_m = -\frac{\dot{m}}{N'} \]

This Hamiltonian correspond to a scalar field with \( m=1 \) in Minkowski spacetime.

- In a natural way, we consider \( \tau \) as the internal time.
The Hamiltonian constraint

\[ H = \partial^2_m - \partial^2_\tau - 1 \]

In terms of the previous variables, the factor ordering gives us the Hamiltonian constraint as

\[ H = \frac{2A}{b^2} \partial^2_A - \frac{2}{b} \partial_A \partial_b + \frac{2}{b^2} \partial_A - \frac{1}{2}(\Lambda b^2 - 1) \]

We choose the kinematical Hilbert space to be the tensor product spanned by

\[ \mathcal{H} = \mathcal{H}_m \otimes \mathcal{H}_\tau \quad \text{with} \quad k_m, k_\tau > 0 \]

where

\[ \phi_{k_m}(m) = e^{\pm ik_m m} \]
\[ \varphi_{k_\tau}(\tau) = e^{\pm ik_\tau \tau} \]
We want to construct our physical Hilbert space coming back to the first variables \((A,b)\)

- The physical observables will be given in terms of these variables
- It is not well defined the position of the horizons in the new variables

We want to define our operators in both regions

- region \(A<0\) → our observable region → \(\Psi_1\)
- region \(A>0\) → beyond the horizons → \(\Psi_2\)
- We can consider the total system, i.e. the region we observe and the regions beyond horizons

- In order to obtain the more general physical states of the system, we will construct the physical Hilbert space as

\[ \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \]

- We have to determine the physical states in both regions

- Observables act in the total system

- As we take \( \tau \) as the internal time, we are interested in identifying this variable in each region, in order to have a common internal time
We could compare it with other isomorphic Hilbert space.

We can consider as our system only the region that we observe and that is classically accessible to us.

Considering the regions independently:

- The observables are defined in each region separately.
- We construct a different Hilbert space for each region, \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \).
Defining a physical observable that allow us to move it along the space → e.g. translation operator

QUANTIZATION OF EACH REGION SEPARATELY

It will be defined in each region uniquely, so we can not move it from one region to another

Like in the classical situation, the regions are totally disconnected and no information can pass through the horizons
The observable will be defined in all regions, thus, we can move the observable for the different regions.

If we trace out the other region, we could see a contribution to our wavefunction coming from the observable in the other region.

We will have in our region a quantum influence of the regions classically disconnected!
Perspectives

- Exact quantization of the Schwarzschild-de Sitter spacetime that allow us to establish **quantum correlations between regions that are not classically accessible**

- Finding a physical observable that mixes regions

- Does this quantization solve the singularity?

- Treating the problem in a more general way with a field theory

- Could we introduce a scalar field?

- Third quantization?