QUANTUM COSMOLOGY: GAMES WITHOUT FRONTIERS

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In classical spacetimes with horizons the different regions are disconnected.

Is it so quantum mechanically?

Open the possibility of quantum connections in the multiverse.
We quantize the system in a manner that allow us to find if these correlations exists

- Minisuperspace model ➔ We develop a canonical quantization
- General spherically symmetric universe ➔ Kantowski-Sachs type with cosmological constant
- Maximal extension ➔ Schwarzschild-de Sitter spacetimes

Quantization of the regions as different subspaces of a same Hilbert space
  - Tensor product or direct sum of the subspaces?
Spherically symmetric **metric** that depends on two variables \((A,b)\)

\[
\sigma^{-2}ds^2 = -\frac{N(r)^2}{A(r)}dr^2 + A(r)dt^2 + b(r)^2d\Omega^2_2
\]

- Generically \(A=0\) corresponds to a horizon
- \(A, b \in \mathbb{R}\)
  - Change of sign in \(A\): radial coordinate from timelike to spacelike
  - Metric invariant under a change of sign in \(b\) \(\Rightarrow\) every trajectory considered twice
It will be convenient for our analysis to introduce

\[ c = Ab \]

Then the **Einstein-Hilbert action**

\[
S = - \int_0^\infty dr \left( \frac{b^2 c}{N} + N \dot{B}(b) \right)
\]

with \[ B(b) = \frac{\lambda}{3} b^3 - b, \quad \dot{B}(b) = \partial_b B(b) = \lambda b^2 - 1 \]

From the point of view of the metric, the solution corresponds to the **Schwarzschild-de Sitter metric**

\[
b(r) = r, \quad A(r) = -1 + 2m/r + \lambda r^2/3
\]
Penrose diagrams ➔ different cases depending on the value of $m$

- $0 < m < \frac{1}{\sqrt{9\lambda}}$
  - $1/\sqrt{9\lambda} < m$
  - $m = 0$
  - $m < 0$
  - $m = \frac{1}{\sqrt{9\lambda}}$
In order to perform a canonical quantization, we are interested in making a Hamiltonian formulation of the system. The canonical action can be expressed as

\[ S = \int_{\mathbb{R}} dr \left( \dot{c} p_c + \dot{b} p_b - NC \right) \]

where the canonical conjugate momenta are

\[ p_b = -\dot{c}/N, \quad p_c = -\dot{b}/N \]

The variation with respect to the lapse function give rise to the Hamiltonian constraint \( NC = 0 \), with

\[ C = -p_b p_c + \dot{B}(b) \]

- Note the symmetry \( (b, c, p_b, p_c) \rightarrow (-b, -c, p_b, p_c) \)
CANONICAL QUANTIZATION

- We follow an extension of **Dirac quantization procedure** [Ashtekar & Tate, 1994]

- Construct a kinematical operator algebra ➔ phase space

- Represent it by operators acting on a **kinematical** complex vector space ➔ Hilbert space structure

- Select the **physical states** by imposing the constraint operator

- **Inner product**: classical dynamic variables be represented as self-adjoint operators
**KINEMATICAL SPACE**

- Kinematical algebra constructed from \((b, c, p_b, p_c)\)

- We choose the vector space spanned by simultaneous solutions to

\[
-i\partial_c \Psi_{hp} = p \Psi_{hp}, \quad [\partial_c \partial_b + \dot{B}(b)]\Psi_{hp} = h \Psi_{hp}
\]

- These solutions have the form

\[
\Psi_{hp}(b, c) = e^{ipc + i[B(b) - bh]/p}
\]

- Kinematical state as a linear combination of these solutions

\[
\Psi(b, c) = \int_{\mathbb{R}} dh \int_{\mathbb{R}} dp \tilde{\Psi}(h, p) \Psi_{hp}(b, c)
\]

- We construct \((b,c)\)-representation and \((h,p)\)-representation
Alternative way to construct the same kinematical space

- Canonical change of variables [Mena Marugán, 1994]

\[
\begin{align*}
t &= -\frac{b}{p_c}, & h &= -p_b p_c + B \\
q &= c - \frac{[B(b) + b p_b p_c - b \dot{B}(b)]}{p_c^2}, & p &= p_c
\end{align*}
\]

where the type-2 generating function

\[
F(c, b, h, p) = cp + \frac{[B(b) - bh]}{p}
\]

- We choose as the kinematical vector space the space of distributions \( \tilde{\Psi}(h, p) \) so

\[
\tilde{h} = h, \quad \tilde{t} = i\partial_h, \quad \tilde{p} = p, \quad \tilde{q} = i\partial_p
\]

- The metric variables can be represented as the operators

\[
\begin{align*}
\hat{b} &= -\hat{t}\hat{p}, & \hat{c} &= \hat{q} + B(-\hat{t}\hat{p})\hat{p}^{-2} + \hat{t}\hat{h}\hat{p}^{-1} \\
\hat{p}_b &= [\hat{B}(-\hat{t}\hat{p}) - \hat{h}]\hat{p}^{-1}, & \hat{p}_c &= \hat{p}
\end{align*}
\]

- We can go to the metric representation by the transformation

\[
\Psi(b, c) = \int_{\mathbb{R}} dh \int_{\mathbb{R}} dp \tilde{\Psi}(h, p) e^{iF(b, c, h, p)}
\]
In terms of these variables \((q, t, p, h) \rightarrow (-q, -t, p, h)\)

It may be convenient to introduce an **inner product** in which the operators \((\hat{h}, \hat{p}, \hat{t}, \hat{q})\) are self-adjoint

\[
(\Psi_1, \Psi_2) = \int_{\mathbb{R}} dh \int_{\mathbb{R}} dp \tilde{\Psi}_1(h, p) \ast \tilde{\Psi}_2(h, p)
\]

so, we choose kinematical Hilbert space as \(L^2(\mathbb{R}^2, dh dp)\)

- The orthonormality of the states \((\Psi_{hp}, \Psi_{h'p'}) = \delta(h - h')\delta(p - p')\)

The **Hamiltonian constraint** in the two representations of this kinematical space

\[
\hat{C} = \partial_b \partial_c + \hat{B}(b), \quad \hat{C} = h
\]
**PHYSICAL SPACE**

- The **space of solutions** can be obtained solving the equation

\[ \hat{C} \Phi(h, p) = h \Phi(h, p) = 0 \]

\[ \Phi(h, p) = \frac{1}{\sqrt{2\pi}} \delta(h) \phi(p) \]

- In these physical states the observables \( \hat{p} = p, \quad \hat{q} = i\partial_p \)

- **Inner product** by imposing that the observables be self-adjoint

\[ \langle \Phi_1, \Phi_2 \rangle = \int_{\mathbb{R}} dp \phi_1(p)^* \phi_2(p) \]

so the physical Hilbert space is \( L^2(\mathbb{R}, dp) \)

- We will refer it as the **p-representation**
The physical states

\[ \Phi(b, c) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} dp \phi(p) e^{i[pc + B(b)/p]} \]

and the inverse

\[ \phi(p) = \frac{1}{\sqrt{2\pi}} e^{-iB(b)/p} \int_{\mathbb{R}} dc \Phi(b, c) e^{-ipc} \]

Inner product in terms of the metric variables

\[ \langle \Phi_1, \Phi_2 \rangle = \int_{\mathbb{R}} dc \Phi_1(b, c)^* \Phi_2(b, c) \]

- Note that the inner product does not depend on b
We have seen that we have a whole family of $c_b$-representations labeled by $b$.

- Inner product

Resembles a kind of transformation from the Heisenberg picture

$$\Phi(b, c) = \hat{U}(b)\Phi(0, c) \quad \text{with} \quad \hat{U}(b) = e^{iB(b)/\hat{p}}$$

- The observables in this representation:

\[ \hat{\pi}_b = -i\hat{\partial}_c, \quad \hat{c}_b = c \]

- The family of observables $\hat{c}_b$ in this representation can be interpreted as giving the value of the variable $c$ at that of $b$.
In the same way we also have a family of $p_b$-representations, given by a Fourier transformation

\[
\phi(b, p) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} dc e^{-ipc} \Phi(b, c) = \hat{U}(b) \phi(p) = \phi(p) e^{iB(b)/p}
\]

- Observables \( \hat{p}_b = p, \quad \hat{c}_b = -i\partial_p \)

- **Schrödinger kind picture**: $p_b$-representation is the evolution of $p$-representation from a value of $B(b)$ that vanishes to a new value of $B(b)$

  \[ \rightarrow \text{Family of representations give the Schrödinger dynamics in } b \]
- **Heisenberg kind picture**: we choose a fixed $p_b$-representation for a given value of $b$, and represent our family of observables

\[ \hat{c}_b^0 = \hat{U}^+(b, b_0) \hat{c}_b \hat{U}(b, b_0) = i \partial_p + \frac{B(b_0) - B(b)}{p^2} \]

where \[ \hat{U}(b, b_0) = e^{i[B(b) - B(b_0)]/\hat{p}} \]

- This observable gives the value of $c$ when $b=b_0$ (in $p_p$-representation)

- Since the function $B$ is not one-to-one, the operators may coincide for some values of $b_0$
We can choose a family of bases, one for each $b$, given by the eigenstates of the self-adjoint operator $\hat{c}_b^0$ in $p_b$-representations with $c_0^0$ eigenvalues

$$\phi_{c^0}(b, p) = \frac{1}{\sqrt{2\pi}} e^{-ipc^0 - i[B(b_0) - B(b)]/p}$$

The equal-$b$ identity operator can be expressed

$$\mathbb{1}(b, p; b, p') = \int_{\mathbb{R}} \text{d}c^0 \phi_{c^0}(b, p) \phi_{c^0}(b, p')^*$$

We can decompose it in the direct sum of two orthogonal projectors

$$\mathbb{1} = \hat{P}_+^0 + \hat{P}_-^0$$

positive $c_0^0$ negative $c_0^0$
If \( b = b_0 \) we observe only the region \( c < 0 \) (classically our region).

Choosing states with null projection under \( \hat{P}_0 \).

- Is this restriction robust?

- Compatibility of measures at different values \( b = b_0, b_1 \)

\[
[\hat{c}_b^0, \hat{c}_b^1] \neq 0
\]

- The eigenstates can not be chosen as common \( \Rightarrow \) projector would not commute
- The measurements would lead to contradictory results
- Restriction to a region of the universe is not stable

\[ b = b_0 \quad \rightarrow \quad \hat{P}^0_+ \Phi = 0 \]

\[ b = b_1 \quad \rightarrow \quad \hat{P}^1_+ \Phi \neq 0 \]

- Depends on the value of \( b \) \( \Rightarrow \) unstable under \( b \)-evolution
If we act with $\hat{c}_b^1$ on the eigenstates of $\hat{c}_b^0$

$$\hat{c}_b^1 \phi_{c^0}(b, p) = \left[ c^0 - \frac{B(b_0) - B(b_1)}{p^2} \right] \phi_{c^0}(b, p)$$

- If $B(b_0) - B(b_1) < 0$ positive $c^0$ in positive $c^1$
- If $B(b_0) - B(b_1) > 0$ negative $c^0$ in negative $c^1$

unless $B(b_0) = B(b_1)$

not satisfied in general

Note that $\hat{q} = i\partial_p$ is a particular case of $\hat{c}_b^0$
- Dynamics in \( b \) mixes the projections

- Hilbert space as a direct sum of subspaces is not consistent

\[ \mathcal{H}^0 = \mathcal{H}^0_+ \oplus \mathcal{H}^0_- \]

- More appropriate to consider general physical states belonging to the tensor product

\[ \mathcal{H}^0 = \mathcal{H}^0_+ \otimes \mathcal{H}^0_- \]

of the projection subspaces \( \mathcal{H}^0_\pm = \hat{P}^0_\pm \mathcal{H} \)
Any observable $\hat{O}$ can be decomposed in four operators between both projection subspaces

\[
\hat{O}^0_{\pm\pm} : \mathcal{H}^0_\pm \rightarrow \mathcal{H}^0_\pm, \quad \hat{O}^0_{\pm\mp} : \mathcal{H}^0_\pm \rightarrow \mathcal{H}^0_\mp
\]

defined as $\hat{O}^0_{\pm\pm} = \hat{P}^0_\pm \hat{O} \hat{P}^0_\pm$, $\hat{O}^0_{\pm\mp} = \hat{P}^0_\pm \hat{O} \hat{P}^0_\mp$

- Operators $\hat{O}^0_{\pm\mp}$ mix the subspaces $\Rightarrow$ cause correlations $\Rightarrow \hat{c}^0_b$

- If the observable is unitary $\Rightarrow$ unitarity is not respected in each subspace separately $\Rightarrow \exp(i\hat{c}^0_b)$

Mixing regions by quantum effects is a generic result in this quantization!
CONCLUSIONS

- Study of the quantum correlations of classically disconnected regions in a single universe
  - Extrapolation to multiverse scenario
- Canonical quantization of the model → physical structure is consistent only if we consider the whole system
  - Mixing regions
  - Unitarity of the global system but not for each subspace
  - Physical states belong to the tensor product of the Hilbert subspaces for each region
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Thank you for your attention!