Precision Top-Quark Mass Measurement at CDF

We present a precision measurement of the top-quark mass using the full sample of Tevatron \( \sqrt{s} = 1.96 \) TeV proton-antiproton collisions collected by the CDF II detector, corresponding to an integrated luminosity of 8.7 fb\(^{-1}\). Using a sample of \( t\bar{t} \) candidate events decaying into the lepton + jets channel, we obtain distributions of the top-quark masses and the invariant mass of two jets from the W boson decays from data. We then compare these distributions to templates derived from signal and background samples to extract the top-quark mass and the energy scale of the calorimeter jets with \textit{in situ} calibration. The likelihood fit of the templates from signal and background events to the data yields the single most-precise measurement of the top-quark mass, \( M_{\text{top}} = 172.85 \pm 0.71(\text{stat}) \pm 0.85(\text{syst}) \) GeV/c\(^2\).

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The top quark \((t)\) is by far the heaviest known elementary particle [1]. It contributes significantly to electroweak radiative corrections relating the top-quark mass \( (M_{\text{top}}) \) and the W boson mass to the mass of the Higgs boson [2]. Precision measurements of \( M_{\text{top}} \) provide therefore important constraints on the Higgs boson mass. Over the last decade, the CDF and D0 experiments have been improving the precision of the \( M_{\text{top}} \) measurement [3], joined recently by the experiments at the Large Hadron Collider [4].

This Letter reports the single most-precise measurement of the top-quark mass to date using the template method [5–8]. In this measurement, we reconstruct top-quark masses in each event and compare the distribution of data with template distributions derived from model calculations to estimate \( M_{\text{top}} \). We also use the template distributions of hadronically decaying W bosons to constrain the jet energy scale, an important uncertainty in the \( M_{\text{top}} \) measurement. This is an update of a previous measurement that used a subset of the present data and determined \( M_{\text{top}} = 172.2 \pm 1.5 \text{ GeV/c}^2 \) [8]. This measurement has an accuracy approximately 30% better than an earlier result with similar template technique and 12% better with respect to the previous best measurement [9]. In this measurement, we use not only larger samples but also improve jet energy calibration using an artificial neural network [10] to achieve better jet energy resolution, and increase signal acceptance allowing less pure signal samples into the analysis. The measurement is performed with \( \sqrt{s} = 1.96 \) TeV Tevatron proton-antiproton collision data collected by the CDF II detector [11] and corresponding to an integrated luminosity of 8.7 fb\(^{-1}\). This is the full Run II CDF data set with requirements of good detector performances.

Assuming unitarity of the quark-mixing matrix [1], the top quark decays almost exclusively into a W boson and a \( b \) particle...
quark. In the $t\bar{t}$ events, the case where one $W$ decays leptonically into an electron ($e$) or a muon ($\mu$) plus a neutrino ($\nu$), including the cascade decay of $W \rightarrow \tau \nu$ and $\tau \rightarrow \mu \nu$ or $\tau \rightarrow e \nu$, and the other $W$ decays hadronically into a pair of jets ($t\bar{t} \rightarrow 4b4q$), defines the lepton + jets decay channel. Lepton + jets events are selected by requiring one isolated [12] electron (muon) with $E_T > 20$ GeV ($p_T > 20$ GeV/$c$) and pseudorapidity $|\eta| < 1.1$ [13]. We also require large missing transverse energy [14], $E_{T}^{miss} > 20$ GeV, and at least four jets. Jets are reconstructed with a cone algorithm [15] with radius $R = \sqrt{\Delta \eta^2 + (\Delta \phi)^2} = 0.4$. In addition to the standard jet energy corrections [16], we train a neural network including additional information to the calorimeter one, such as jet momentum from the tracker as described in Ref. [10]. We have performed the training separately for $b$ quarks and light flavor quarks from $W$ boson decay. The additional information on the jets improves the resolution of the jets as well as the reconstructed top-quark masses and hadronically decaying $W$ boson mass. This allows approximately 13% better statistical precision compared with standard correction.

Jets originating from $b$ quarks are identified (tagged) using a secondary-vertex tagging algorithm [17]. We divide the sample of candidate lepton + jets events into subsamples based on the number of identified $b$ jets, zero $b$-tagged jet (0-tag), one $b$-tagged jet (1-tag), and two or more $b$-tagged jets (2-tag). In the 0-tag events, we require exactly four tight jets (transverse energy $E_T > 20$ GeV and $|\eta| < 2.0$). In the 1-tag and 2-tag events, three tight jets and at least one loose jet ($E_T > 12$ GeV and $|\eta| < 2.4$) are required. We divide the 1-tag and 2-tag samples into subsamples based on the number of tight jets and call the “tight” subsample the one requiring exactly four tight jets and “loose” the one consisting of the remaining events passing selection. The measurement uses five subsamples (0-tag, 1-tagL, 1-tagT, 2-tagL, and 2-tagT, where L and T represent loose and tight selection, respectively). The subsamples of 0-tag and 1-tagL are newly added in this measurement. The introduction of these high-background subsamples offers a gain of approximately 12% in the statistical precision. We apply an additional requirement on the scalar sum of transverse energies in the event, $H_T = E_T^{lepton} + E_T^{miss} + \sum_{jets} E_T^{jet}$, to be greater than 250 GeV for 0-tag and 1-tag events, where $E_T^{lepton}$ is electron or muon transverse energy or momentum, respectively, and $E_T^{jet}$ is the transverse energy of the jet. This requirement is not applied to the 2-tag events because of small background contribution in these subsamples.

The primary background sources are $W +$ jets and QCD multijet production. We also consider small contributions from $Z +$ jets, diboson, and single-top-quark production. To estimate the contribution of each process, we use a combination of data- and Monte Carlo (MC)-based techniques described in Ref. [18]. For the $Z +$ jets, diboson, single top, and $t\bar{t}$ events we normalize simulated events using their theoretical cross sections [19–21]. We use the data-driven techniques described in Ref. [22] to estimate QCD multijet background. The shape of the $W +$ jets background is obtained from simulation, while the number of $W +$ jets events is determined from the total number of events in data minus the estimate for the other backgrounds and $t\bar{t}$ event contributions.

For each event, three observables are used, two reconstructed top-quark masses [$m_{reco}^{t}$ and $m_{reco}^{t(2)}$] and the invariant mass of the two jets from the hadronically decaying $W$ boson ($m_{jj}$). We have a complete reconstruction of the $t\bar{t}$ kinematics in the lepton + jets channel [5,6] with constraints from the precisely known $W$ boson mass and requiring the $t$ and $\bar{t}$ masses to be the same. Assuming that the leading four jets in the detector originate from the $t\bar{t}$ decay products, there are twelve, six, and two assignments of jets to quarks for 0-tag, 1-tag, and 2-tag events, respectively. A minimization is performed for each assignment using a $\chi^2$ comparison to the $t\bar{t}$ hypothesis with $m_{reco}^{t}$ taken from the assignment that yields the lowest $\chi^2 (\chi^2_{min})$. To reject poorly reconstructed events, we require $\chi^2_{min} < 3$ and $\chi^2_{min} < 9$ for 0-tag and tagged (both 1-tag and 2-tag) events, respectively. To increase the statistical power of the measurement, we employ an additional observable $m_{reco}^{t(2)}$ from the assignment that yields the second lowest $\chi^2$. The dijet mass $m_{jj}$ is calculated as the invariant mass of two non–$b$-tagged jets that provides the closest value to the known $W$ boson mass of 80.39 GeV/$c^2$ [23]. We apply boundary conditions on $m_{reco}^{t}$ and $m_{reco}^{t(2)}$ [$100 < m_{reco}^{t}$, $m_{reco}^{t(2)} < 350$ GeV/$c^2$] and also $m_{jj}$ ($60 < m_{jj} < 110$ GeV/$c^2$, $50 < m_{jj} < 120$ GeV/$c^2$, and $50 < m_{jj} < 125$ GeV/$c^2$ for 0-tag, 1-tag, and 2-tag events, respectively). The estimated number of background events and the observed numbers of events after event selection, $\chi^2$, and boundary requirements are listed in Table I.

We estimate the probability density functions (PDFs) of signal and background using a kernel density estimation method [24]. A three-dimensional kernel density estimation [8] accounts for the correlation between the three observables. The dijet mass $m_{jj}$ of the two jets assigned to the $W$ in the lepton + jets channel is used for in situ calibration of the jet energy scale (JES) [5,6]. The PDFs for the observables are estimated at 76 discrete values of $M_{top}$ from 130 GeV/$c^2$ to 220 GeV/$c^2$ and at 29 discrete values of $\Delta_{JES}$ from $-3.0\sigma_c$ to $3.0\sigma_c$, where $\sigma_c$ is the CDF JES fractional uncertainty based on a combination of instrumental calibration and analysis of data control samples [16]. The parameter $\Delta_{JES}$ determines the correction factor of the jet energies by a factor of $1 + \Delta_{JES}$ [5,6]. We interpolate the MC distributions to find PDFs for arbitrary values of $M_{top}$ and $\Delta_{JES}$ using the local polynomial smoothing method [25]. We fit the signal and background PDFs to the distributions of the observables in the data using an unbinned maximum likelihood fit [26,27].
Independent likelihoods are used for each subsample, 0-tag, 1-tagL, 1-tagT, 2-tagL, and 2-tagT, and the total likelihood is obtained by multiplying them together [6–8]. References [6–8] provide detailed information about this technique.

We test the mass determination using 1500 statistical trials for a set of 11 different $M_{\text{top}}$ values ranging from 160 GeV/c$^2$ to 185 GeV/c$^2$. In each experiment, we draw the number of signal and background events from a Poisson distribution centered at the expected number of signal and total background shown in Table I, respectively. The distributions of the average deviation from the input top mass and the width of the deviation normalized to the estimated uncertainty for simulated experiments are corrected to be unity and zero, respectively. The correction is $M^{\text{corr}}_{\text{top}} = 1.03 \times M^{\text{meas}}_{\text{top}} - 4.88$ GeV/c$^2$, where $M^{\text{meas}}_{\text{top}}$ is the maximum likelihood estimate and $M^{\text{corr}}_{\text{top}}$ is the corrected value of the measurement. We increase the measured uncertainty by 2.9% to correct the width of the pull. We also test the mass fit results using different values of $\Delta_{\text{JES}}$ between $-1.0\sigma_c$ to $1.0\sigma_c$ with three different $M_{\text{top}}$ points, 168, 173, and 178 GeV/c$^2$. With a correction discussed above, the residuals of $M_{\text{top}}$ from different $\Delta_{\text{JES}}$ values are consistent with zero.

We examine various sources of systematic uncertainties by comparing the analysis results of statistical trials in which we vary relevant parameters within their uncertainties. The dominant sources are the residual JES [16] and signal modeling. We vary the JES parameters within their uncertainties in both signal and background MC generated events and interpret the deviations of the results as additional uncertainties. The uncertainty arising from the choice of MC generator (signal modeling) is estimated by comparing the results of pseudoexperiments generated with PYTHIA [28] and HERWIG [29]. We examine the effects of higher-order corrections using MC@NLO [30], a full next-to-leading-order simulation. The systematic uncertainty to the energy corrections of $b$ jets ($b$-JES) arising from our modeling of $b$ fragmentation, $b$ hadron branching fractions, and calorimeter response, captures the additional uncertainty not taken into account in the residual JES.

The uncertainty on the $b$-tagging efficiency can propagate into a bias of the $M_{\text{top}}$ measurement which is taken as a systematic uncertainty. The uncertainty due to the limited knowledge of initial-state radiation is constrained by studies of radiation in Drell-Yan events. We vary both initial and final state radiation within these constraints by extrapolating in the $p_T$ of Drell-Yan events to the $t\bar{t}$ mass region [5]. We estimate the systematic uncertainty due to parton distribution functions by varying the independent eigenvectors of the CTEQ6M [31] parton distribution functions, varying the QCD scale ($\Lambda_{\text{QCD}}$), and comparing our nominal CTEQ5L [32] with MRST72 [33] parton distribution functions. We vary the gluon fusion fraction from 5% to 20%, corresponding to the 1 standard deviation upper bound on the gluon fusion fraction [34]. We estimate systematic uncertainties due to the lepton energy and momentum scales by propagating shifts in electron energy and muon momentum scales within their uncertainties. The background shape systematic uncertainty accounts for the variation of the background composition. We estimate a multiple-hadron-interaction systematic uncertainty to account for the difference in the average number of interactions.

<table>
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<tr>
<th>Source</th>
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<tr>
<td>Residual jet energy scale</td>
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</table>
The distributions of the observables used for the full background model.

Figure 1 shows the distributions of the observables used to measure $M_{\text{top}}$ for 0-tag and tagged events. The data are overlaid with the predictions from the kernel density estimation probability distributions using $M_{\text{top}} = 173$ GeV/c² and the full background model.

Interactions between simulation and data. The color reconnection systematic uncertainty [35] is evaluated using samples simulated with and without color reconnection effects with different PYTHIA tunes [36]. All systematic uncertainties are summarized in Table II. The total systematic uncertainty adding individual components in quadrature is 0.85 GeV/c². The details of systematic uncertainty evaluations are in Ref. [3,5,6].

We perform the likelihood fit to the data and apply the corrections obtained using the simulated experiments, and measure

$$M_{\text{top}} = 172.85 \pm 0.71(\text{stat}) \pm 0.85(\text{syst}) \text{ GeV/c}^2.$$  

Figure 1 shows the distributions of the observables used for the $M_{\text{top}}$ measurement in the lepton + jets channel, overlaid with density estimates using $t\bar{t}$ signal events with $M_{\text{top}} = 173$ GeV/c² (close to the measured $M_{\text{top}}$) and the full background model.

In conclusion, we have performed a measurement of the top-quark mass using the template method in the lepton + jets using the full CDF Run II data set corresponding to $8.7 \text{ fb}^{-1}$ $p\bar{p}$ collisions. The result, $M_{\text{top}} = 172.85 \pm 1.11 \text{ GeV/c}^2$, is the best single measurement of this important physics parameter. It is consistent with the most recent Tevatron average of $M_{\text{top}} = 173.18 \pm 0.94 \text{ GeV/c}^2$ [3] and will significantly contribute to the Tevatron’s and world’s average value for the top-quark mass.

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A lepton is isolated when \( \left( p_T^{\text{lepton}} - p_T^\ell \right) / p_T^\ell < 0.1 \), where \( p_T^\ell \) is the total transverse momentum (energy) and \( p_T^{\text{lepton}} \) is the lepton transverse momentum (energy) for a muon (electron) in a cone of radius \( \Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = 0.4 \) with axis along the direction of the lepton.

We use a right-handed spherical coordinate system with the origin at the center of the detector with the proton beam and the \( y \) axis pointing up. The pseudorapidity is defined by \( \eta = -\ln \tan(\theta/2) \). The transverse momentum and energy of a detected particle or jet are defined by \( p_T = p \sin(\theta) \) and \( E_T = E \sin(\theta) \), respectively, where \( p \) and \( E \) are the momentum and energy of the particle.

The missing transverse energy, an imbalance of energy in the transverse plane of the detector, is defined by \( E_T = \sum p_T \hat{\eta} \), where \( \hat{\eta} \) is the unit vector normal to the beam and pointing to a given calorimeter tower and \( E_T \) is the transverse energy measured in that tower.


